ZIAUDDIN UNIVERSITY
EXAMINATION BOARD

EXAMINATION MATERIAL ZUEB - 2022

MATHEMATICS XI

SECTION "B" CONSTRUCTED RESPONSE QUESTION (CRQ'S)

CHAPTER NO 1	REAL AND COMPLEX NUMBER SYSTEM

EXERCISE 2.2

Q1. Solve the following complex equations:

i. $(x, y)(2, 3) = (-4, 7)$

ii. $(x + 3i)^2 = 2yi$

iii. $(x+2yi)^2 = xi$

iv. $(-x, 3y) = (2, 0)$

Q2. Evaluate:

i. $|5z_1 - 4z_2|$

ii. $(z_1)^2$

iii. $\frac{z_1}{z_2}$

Where $z_1 = 1 + i$ and $z_2 = 3 - 2i$

Q3. Verify that:

i. $(\sqrt{2} - i) + i(\sqrt{2}\, i - 1) = -2i$

ii. $(1 - i)^4 = -4$

iii. $i^3 = -i \text{ and } i^4 = 1$

iv. $\frac{1+2i}{3-4i} + \frac{2}{5} = \frac{i-2}{5i}$

EXERCISE 2.3

Q4. Find the real and imaginary parts of:

i. $i(3 + 2i)$

ii. $\frac{2-i}{3i}$

iii. $\frac{3a+2bi}{a-bi}$

iv. $\frac{\sqrt{3}+i}{\sqrt{3}-i}$

v. $(a + 3\,bi)^4$

EXERCISE 3.3

Q4. Find all cube roots of:

i. -8
ii. 27
iii. -64
iv. 729

Q5. Prove that:

i. $\omega^{32} + \omega^{37} = -1$
ii. $(1 - \omega - \omega^2)^5 = 32$
iii. $(7 + \omega)(7 + \omega^2) = 43$
iv. $(1 + \omega)^7 = -\omega^2$

EXERCISE 3.5

Q6. Determine the nature of roots of each of the following equations:

i. $x^2 - 2x + 5 = 0$
ii. $2x^2 + 9 = 9x$

Q7. Determine the value of *k* in each of the following equations that will make the roots equal.

i. $x^2 - 2x\,(1 + 3k) + 7\,(3 + 2k) = 0$
ii. $9y^2 + ky + 16 = 0$
iii. $(k + 1)x^2 + 2\,(k + 3)x + (2k + 3) = 0, provided\ k \neq -1.$

Q8. Show that the roots of:

$(x - p)(x - q) + (x - q)(x - r) + (x - r)(x - p) = 0$

are real and they cannot be equal unless $p = q = r$.

Q9. For what values of p and q will both roots of the equation:

$y^2 + (2p - 4)y = 3q + 5$, vanish?

EXERCISE 3.6

Q10. Find k if one root of $4y^2 - 7ky + k + 4 = 0$ is zero**.**

Q11. Find m if the sum of the roots of $6z^2 - 3mz + 5 = 0$ is equal to the product of its roots.

EXERCISE 3.8

Q12. Solve the following systems of equation:

i. $x + y = 5$

$\frac{3}{x} + \frac{2}{y} = 2$

ii. $y + z = 5$

$y^2 + 2z^2 = 17$

iii. $xt + 15 = 0$

$x^2 + t^2 = 34$

iv. $2x^2 + xy + y^2 = 8$

$6xy + 2y^2 = 20$

CHAPTER NO 4	MATRICES & DETERMINANTS

EXERCISE 4.1

Q13. Wherever possible, find matrix X so that:

i. $\begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$, X= $\begin{bmatrix} -2 & 5 \\ 8 & -7 \end{bmatrix}$

ii. X $\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q14. Solve for *x*:

i. $\begin{bmatrix} -2 & 3 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 1 & x & 5 \\ 2 & 4 & x \end{bmatrix} . \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 & -14 \end{bmatrix}$

Q15. Find *x* , *y*, *z* and *v* so that:

i. $\begin{bmatrix} 4 & x+y \\ z+v & 3 \end{bmatrix} = 3\begin{bmatrix} x & y \\ z & v \end{bmatrix} + \begin{bmatrix} x & 6 \\ -1 & 2v \end{bmatrix}$

Q16. Perform the matrix multiplication:

i. $\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

ii. $\begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 \\ i & 0 \end{bmatrix} . \begin{bmatrix} 1 & i \\ -i & 0 \end{bmatrix}$; $(i = \sqrt{-1})$

Q17. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ $.\, show\ that$: $A^2 - 4A - 5I_3 = O_3$.

Q18. Prove the following identities:

$\left\{ \begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{bmatrix} + \begin{bmatrix} \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{bmatrix} \right\} \begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$; where ω is a complex cube root of unity.

EXERCISE 5.1

Q19. Let S= {A, B, C, D}, where A={a}, B= {a, b}, C= {a, b, c} and D = ∅.

Construct multiplication tables to show that $\cup$ and $\cap$ are binary operations on S.

Q20. Show that multiplication is a binary operation on S = {1, -1, i, -i }, where $i = \sqrt{-1}$ is multiplication commutative and associative in S.

Q21. **Define a binary operation $*$ $in\ \varphi$ by** $a * b = 4a\,.b, \quad \forall\ a, b \in \varphi$

Where "." Represents ordinary multiplication.

Show that:

i. * is commutative.

ii. * is associative.

iii. $\frac{1}{4}$ is the identity element w.r.t *.

iv. $\frac{1}{12}$ is the inverse of $\frac{3}{4}$ under *.

Q22. Let $S = \{1, \omega, \omega^2\}, \omega$ being a complex cube root of unity. Construct a composition table w.r.t multiplication on $\mathbb{C}$ and show that:

i. Associative law holds.

ii. 1 is the identity element in S.

iii. Each element of S has an inverse in S.

Q23. Let * be defined in Z by: $m * n = m + n + 2$

i. Show that * is associative and commutative.

ii. Identity w.r.t * exists in Z.

iii. Every element of Z has an inverse under *.

EXERCISE 6.1

Q24. How many terms are there in the A.P. $\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \dots\dots$, whose last term is $-\frac{17}{6}$?

EXERCISE 6.2

Q25. Find the sum of all natural numbers between 250 and 1000 which are exactly divisible by 3 or 7.

Q26. Show that the sum if the first n even natural numbers is equal to $\left(1+\frac{1}{n}\right)$ times the sum of the first n odd natural numbers.

EXERCISE 6.4

Q27. Find the G.P in which:

i. $T_3 = 10\ and\ T_5 = 2\frac{1}{2}$

ii. $T_5 = 8\ and\ T_8 = -\frac{64}{27}$

Q28. A ping pong ball is dropped vertically on a table from a height of 81 cm above the table. It always bounces back two third-of the distance of the previous fall. How high does it bounce back after striking the table for the 4^{th} time?

EXERCISE 6.5

Q29. Find the sum of the first n terms of the following series:

i. 7 + .77 + .777 + …

ii. 6 + 66 + 666 + …

EXERCISE 6.8

Q30. The 12^{th} term of an H.P is $\frac{1}{5}$ and the 19^{th} term is $\frac{3}{22}$. Find the 4^{th} term.

Q31. **Insert:**

i. A single H.M between $\frac{1}{2}$ and $\frac{1}{3}$

ii. Four H.M.'s between 12 and $\frac{48}{5}$.

CHAPTER NO 7 PERMUTATIONS, COMBINATION & INTRODUCTION TO PROBABILITY

EXERCISE 7.2

Q32. Find n, if:

i. ${}^{n}P_2 = 20$

ii. ${}^{2n}P_3 = 2.({}^{n}P_4)$

Q33. How many different arrangements can be made by using all the letters of the word "MATHEMATICS"? How many of them begin with "C"? How many of them begin with "T"? In how many of them consonants will occur together?

Q34. Four out of 10 balls are red, some are green, and the rest are of different colors. If the balls can be arranged in 6300 ways, find the number of green balls.

Q35. In how many ways can 3 English, 2 Urdu and 2 Sindhi books can be arranged on a shelf so as to keep all the books in each language together?

CHAPTER NO 8 MATHEMATICAL INDUCTION & BINOMIAL THEOREM

EXERCISE 8.1

Q36. Prove the following propositions by mathematical induction:

(i) $2 + 6 + 12 + \dots + n(n+1) = \frac{1}{3} n (n+1) (n+2)$

(ii) $1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{1}{3} n (2n-1)(2n+1)$

(iii) $1.3 + 2.4 + 3.5 + \dots + n (n+2) = \frac{1}{6} n (n+1) (2n+7)$

(iv) $1.2.3 + 2.3.4 + 3.4.5 + \dots + n (n+1) (n+2) = \frac{1}{4} n (n+1) (n+2) (n+3)$

Q37. **Prove the following propositions by mathematical induction:**

i. $2^{3n+2} - 28n - 4$ is divided by 49, $\forall n \in N$

ii. $11^{n+2} + 12^{2n+1}$ is divided by 133 for all integral values of $n \geq 0$.

iii. $a^{2n} - b^{2n}$ is divisible by a+b for all $n \in N$

EXERCISE 8.3

Q38. Write, in the simplified form, the term independent of x in:

i. $(x - \frac{2}{x})^{10}$ ii. $(\sqrt{x} - \frac{2}{x^2})^{10}$

Q39. Obtain in the simplified form:

i. The term involving x^2 in the expansion of $(2x - \frac{1}{2})^7$

ii. The coefficient of y in the expansion of $(y^2 + \frac{a^3}{y})^5$

iii. The coefficient of y^{-2} in the expansion of $(y + \frac{b^3}{y^2})^{10}$

EXERCISE 8.5

Q40. Identify the following series as binomial expansions and find their sums:

i. $1 + \frac{2}{3} . \frac{1}{2} + \frac{2.5}{3.6} . \frac{1}{2^2} + \frac{2.5.8}{3.6.9} . \frac{1}{2^3} + \cdots$

ii. $1 - \frac{3}{4} + \frac{3.5}{4.8} - \frac{3.5.7}{4.8.12} + \cdots$

iii. $\sqrt{3} = 1 + \frac{1}{3} + \frac{1.3}{3^2.2!} + \frac{1.3.5}{3^3.3!} + \cdots$

CHAPTER NO 9 FUNDAMENTAL OF TRIGONOMETRY

EXERCISE 9.1

Q41. How far does a boy on bicycle travel in 10 revolutions if the diameters of the wheels of his bicycle each equal to 56cm.

Q42. If a point on the rim of a 21cm. diameter fly wheel travels 5040 meters in a minute, through how many radians does the wheel turn in a second?

Q43. A belt 24.75 meters long passes around a 1.5cm diameter pulley. As the belt makes two complete revolutions in a minute. How many radians does the wheel turn in one second?

Q44. A car is running on a circular path of radius equal to double the arc of the circle travelled by the car. Find the angle subtended by the arc at the center of circular path.

EXERCISE 9.2

Q45. Find the remaining trigonometric function in the following, if:

i. $sin\theta = \frac{\sqrt{3}}{2}$ and $\rho(\theta)$ is in the second quadrant.

ii. $cosec\theta = -\frac{3}{2}$ and $\rho(\theta)$ is not in the fourth quadrant.

iii. $cot\theta = 3$ and $sin\theta$ is positive.

iv. $tan\theta = -\frac{1}{3}$ and $sin\theta$ is negative.

CHAPTER NO 10 TRIGONOMETRIC IDENTITIES

EXERCISE 10.2

Q46. Prove that:

i. $\frac{\sin(\theta-\emptyset)}{\cos\theta\ \cos\emptyset} = \tan\theta + \tan\emptyset$ (when $\cos\theta\ \cos\emptyset\ \neq 0$)

ii. $\cos(\theta+\emptyset)\cos(\theta-\emptyset) = \cos^2\theta - \sin^2\emptyset$

iii. $\sin(\theta+\emptyset)\sin(\theta-\emptyset) = \sin^2\theta - \sin^2\emptyset$

iv. $\cos(\theta+\emptyset) - \cos(\theta-\emptyset) = -2\sin\theta\sin\emptyset$

v. $\sin(\theta+\emptyset) + \sin(\theta-\emptyset) = -2\sin\theta\cos\emptyset$

CHAPTER NO 11 GRAPHS OF TRIGONOMETRIC FUNCTION

EXERCISE 11.2

Q47. Draw the graph of $\sin\theta, where -\pi \leq \theta \leq \pi$. From the graph, find the value of sin 130°

Q48. Draw the graph of $\cos 2\theta$, where $-\pi \leq \theta \leq \pi$

CHAPTER NO 12 SOLUTION & TRIANGLES

EXERCISE 12.2

Q49. Solve the following triangle:

i. $\alpha = 49°, \beta = 60°, c = 39cm$

Q50. A hiker walks due east at 4 km per hour and a second hiker, starting at the same point, walks 55° north east at the rate of 5km per hour. How far a part will they be after 3 hours?

Q51. A piece of plastic strip 1 meter long is bent to form an isosceles triangle with 95° as measure of its largest angle. Find the length of the sides.

EXERCISE 12.5

Q52. If a = b = c, then prove that**: r_1: R: r = 3:2:1**

Q53. Show that in any ΔABC:

i. $\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca} = \frac{1}{2r\ R}$

ii. $r_1 r_2 r_3 = rs^2$

iii. $r r_1 r_2 r_3 = \Delta^2$

iv. $\frac{1}{r^2} + \frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} = \frac{a^2+b^2+c^2}{\Delta^2}$

Q54. Show that $r_1 = a\frac{\cos\frac{\beta}{2}\cos\frac{\gamma}{2}}{\cos\frac{\alpha}{2}}$

CHAPTER NO 13 INVERSE TRIGONOMETRIC FUNCTION & TRIGONOMETRIC EQUATIONS

EXERCISE 13.2

Q55. Solve:

i. $\sqrt{3}\cos\theta + \sin\theta - 2 = 0$
ii. $\tan^2\theta + \tan\theta = 2$
iii. $4\sin^2\theta\tan\theta + 4\sin^2\theta - 3\tan\theta - 3 = 0$

Extra Question (Derivations)

Q56.

i. Derive the law of sine
ii. Derive the law of cosine
iii. Derive the law of tangent
iv. Prove that: $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$
v. Prove that: $R = \frac{abc}{4\Delta}$