ZIAUDDIN UNIVERSITY
EXAMINATION BOARD

Mathematic
X- GEN
Student Resource

Contents

ADDITION AND SUBTRACTION OF SURDS

There is no simple way to express the sum $\sqrt{2} + \sqrt{3}$ in simpler form. These two surds are called **unlike surds**, in much the same way we call 2*x* and 3*y* unlike terms in algebra. On the other hand $5\sqrt{7}$ and $3\sqrt{7}$ are **like surds**. We can simplify the sum $5\sqrt{7} + 3\sqrt{7}$ to $8\sqrt{7}$, since we can simply think of it as

'5 lots of $\sqrt{7}$ plus 3 lots of $\sqrt{7}$ equals 8 lots of $\sqrt{7}$.

Thus, **we can only simplify the sum or difference of like surds**.

When dealing with expressions involving surds, it may happen that we are dealing with surds that are unlike, but which can be simplified to produce like surds. Thus, we should simplify the surds first and then look for like surds.

EXAMPLE

Simplify $\sqrt{27} + 2\sqrt{5} + \sqrt{20} - 2\sqrt{3}$.

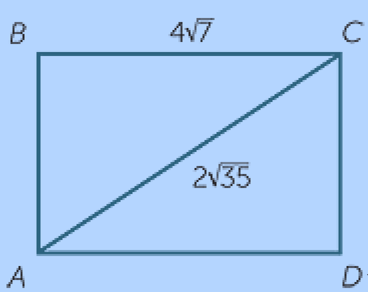
SOLUTION

Simplifying first, we obtain

$$\begin{aligned}\sqrt{27} + 2\sqrt{5} + \sqrt{20} - 2\sqrt{3} &= 3\sqrt{3} + 2\sqrt{5} + 2\sqrt{5} - 2\sqrt{3}\\ &= \sqrt{3} + 4\sqrt{5}.\end{aligned}$$

This expression cannot be simplified further.

EXERCISE 2



In the diagram, find *BA* and the perimeter of the rectangle in surd form.

MULTIPLICATION AND DIVISION OF SURDS

When we come to multiply two surds, we simply multiply the numbers outside the square root sign together, and similarly, multiply the numbers under the square root sign, and simplify the result. A similar procedure holds for division.

- $a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$
- $a\sqrt{b} \div c\sqrt{d} = \frac{a\sqrt{b}}{c\sqrt{d}}$

EXAMPLE

Find **a** $4\sqrt{14} \times 2\sqrt{2}$ **b** $15\sqrt{35} \div 5\sqrt{7}$

SOLUTION

a $4\sqrt{14} \times 2\sqrt{2} = 8\sqrt{28}$

$= 16\sqrt{7}$

b $15\sqrt{35} \div 5\sqrt{7} = 3\sqrt{5}$

THE DISTRIBUTIVE LAW AND SPECIAL PRODUCTS

The usual rules of algebra also, hold when pronumerals are replaced by surds.

For example the identities,

$a(b + c) = ab + ac$, $(a + b)(c + d) = ac + ad + bc + bd$ are useful.

After expanding, it will often be necessary to simplify the surds as well.

EXAMPLE

Expand and simplify

a $2\sqrt{3}(4 + 3\sqrt{3})$ **b** $(3\sqrt{2} - 4\sqrt{3})(5\sqrt{3} - \sqrt{2})$

SOLUTION

a $2\sqrt{3}(4 + 3\sqrt{3}) = 8\sqrt{3} + 6\sqrt{9} = 8\sqrt{3} + 18$

b $(3\sqrt{2} - 4\sqrt{3})(5\sqrt{3} - \sqrt{2}) = 15\sqrt{6} - 6 - 60 + 4\sqrt{6}$

$= 19\sqrt{6} - 66$

DIFFERENCE OF SQUARES

A very important special identity is the difference of squares.

$$(a - b)(a + b) = a^2 - b^2$$

EXAMPLE

Expand and simplify $(2\sqrt{5} - 3\sqrt{2})(2\sqrt{5} + 3\sqrt{2})$.

SOLUTION

$(2\sqrt{5} - 3\sqrt{2})(2\sqrt{5} + 3\sqrt{2}) = (2\sqrt{5})^2 - (3\sqrt{2})^2 = 20 - 18 = 2$.

Notice in the above example, since we are taking a difference of squares, the answer turns out to be an integer. We will exploit this idea in the next section.

EXERCISE 3

Caculate $(5\sqrt{3} + 7\sqrt{2})(5\sqrt{3} - 7\sqrt{2})$.

SQUARES

In addition to the important difference of two squares mentioned above, we also have the algebraic identities:

$(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$,

that also, of course, apply to surds.

EXAMPLE

Expand and simplify

a $(5\sqrt{2} + 3\sqrt{3})^2$ **b** $(3\sqrt{2} - \sqrt{10})^2$

SOLUTION

a $(5\sqrt{2} + 3\sqrt{3})^2 = 25\sqrt{4} + 9\sqrt{9} + 30\sqrt{6} = 77 + 30\sqrt{6}$.

b $(3\sqrt{2} - \sqrt{10})^2 = 9\sqrt{4} + 10 - 6\sqrt{20} = 28 - 12\sqrt{5}$.

UNIT 5

Remainder Theorem and Factor Theorem

SLO'S:

- State and apply remainder theorem.
- Calculate remainder (without dividing) when a polynomial is divided by a linear polynomial.
- Define zeros of a polynomial.
- State factor theorem and explain through examples

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 2

PTBB Math class 9 Chapter 5

STBB Math 9 & 10 Chapter 5

UNIT 6

Factorization of Cubic Polynomial

SLO'S:

- Apply factor theorem to factorize a cubic polynomial.

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 2

PTBB Math class 9 Chapter 5

STBB Math 9 & 10 Chapter 5

UNIT 7

Highest Common Factor (HCF)/Greatest Common Divisor (GCD) and Least Common Multiple (LCM)

SLO'S:

- Calculate Highest Common Factor (HCF) and Least Common Multiple (LCM) of algebraic expressions by factorization method.
- Apply division method to determine highest common factor and least common multiple.
- Describe the relationship between HCF and LCM.
- Solve real life problems related to HCF and LCM.

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 3

STBB Math 9 & 10 Chapter 5

PTBB Math class 9 Chapter 6

UNIT 8

Basic Operations on Algebraic Fractions

SLO'S:

- Apply highest common factor and least common multiple to reduce fractional expressions involving addition(+), subtraction(-), multiplication(and division(

VIDEOS

REFERENCES:

NO

UNIT 9

Square Root of an Algebraic Expression

SLO'S:

- Calculate square root of an algebraic expression by factorization and division methods.

VIDEOS

REFERENCES:

NO

UNIT 10

Linear Equations

SLO'S:

- Define linear equations in one variable.
- Solve linear equations with rational coefficients.
- Convert equations, involving radicals, to simple linear form and calculate their solutions and its verification.
- solve word problems based on linear equation and verify its solutions.

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 4

PTBB Math class 9 chapter 7

UNIT 11

Equations involving Absolute Values

SLO'S:

- Define absolute value.
- Solve the equations, involving absolute values in one variable.

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 4

PTBB Math class 9 chapter 7

UNIT 12

Linear Inequalities

SLO'S:

- Define inequalities (> ,<) and (.
- Describe the properties of inequalities (i.e. trichotomy, transitive, additive and multiplicative

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 4

PTBB Math class 9 chapter 7

UNIT 13

Solving Linear Inequalities

SLO'S:

- Solve linear inequalities with real coefficient, in one variable.
- Represent the solution of linear inequalities on the number line.
- Solve linear inequalities, involving absolute value, in one variable of the following cases on the number line:

  a) 1 x 1 < 0

  b) 1 x 1 > 0

  c) 1 x 1 < 1

  d) 1 x 1 > 1

  e) 1 x 1 < 0, Where a is an integer.

  f) 1 x a 1 > 0 , Where a is an integer.

- Represent the solution of the above cases on the number line.

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 4

PTBB Math class 9 chapter 7

UNIT 14

Quadratic Equations

SLO'S:

- Elucidate, then define quadratic equation in its standard form.

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 5

UNIT 15

Solution of Quadratic Equation

SLO'S:

- Solve a quadratic equation in one variable by Factorization.
- Solve a quadratic equation in one variable by Completing the squares.

REFERENCES:

PTBB General Math class 10 Chapter 5

UNIT 16

Quadratic Formula

SLO'S:

- Apply method of completing the squares to drive the quadratic formula.
- Apply quadratic formula to solve quadratic equations.
- Solve simple real life problems involving related to quadratic formula.

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 5

PTBB General Math class 10 Chapter 5

UNIT 17

Introduction to Matrices

SLO'S:

- Define of the following terms:
- A matrix with real entries and relate its rectangular layout (formation) with representation in real life as well.
- The rows and columns of a matrix.
- The order/size of a matrix.
- Equality of two matrices.

REFERENCES:

PTBB General Math class 10 Chapter 6

STBB General Math book Class 9-10 Chapter 9

UNIT 18

Types of Matrices

SLO'S:

- Define and identify of the followings:

a) Row matrix.

b) Column matrix.

c) Rectangular matrix.

d) Square matrix.

e) Zero/Null matrix.

f) Identity/Unit matrix.

g) Scalar matrix.

h) Diagonal matrix.

i) Transpose of a matrix.

j) Symmetric (upto three by three, 3 x 3).

- Skew-Symmetric matrices.

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 6

STBB General Math book Class 9-10 Chapter 9

UNIT 19

Addition and Subtraction of Matrices

SLO'S:

- Define whether the given matrices are conformable for addition and subtraction.
- Add and subtract matrices.
- Scalar multiplication of a matrix by a real number.
- Verify commutative and associative laws with respect to addition.
- Explain additive identity of a matrix.
- Calculate additive inverse of a matrix.

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 6

STBB General Math book Class 9-10 Chapter 9

UNIT 20

Multiplication of Matrices

SLO'S:

- Describes whether the given matrices are conformable for multiplication.
- Multiply two (or three) matrices.
- Verify associative law under multiplication.
- Prove that distributive laws.
- Prove that with the help of an example that commutative law with respect to multiplication does not hold, in general. (i.e., $AB \neq BA$)
- Define multiplicative identity of a matrix.

- Verify the result (A =

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 6

PTBB General Math class 10 Chapter 6

STBB General Math book Class 9-10 Chapter 9

UNIT 21

Determinant of a Matrix

SLO'S:

- Define the determinant of square matrix.
- Evaluate determinant of matrix.
- Describe the followings:

Singular and Nonsingular matrices.

- Define adjoint of matrix.

·Calculate the multiplicative inverse of a non-singular matrix ' A' and verify that A−1 = I = −1A , where, I is the identity matrix.

- Apply the adjoint method to calculate inverse of a non- singular matrix.

·Prove that (A)−1 = −1−1 . by the help an example.

VIDEOS

REFERENCES:

STBB General Math book Class 9-10 Chapter 9

UNIT 22

Solution of Simultaneous Linear Equations

SLO'S:

- Solve the system of two linear equations, related to real life problems, in two unknowns using:

a) Matrix Inversion Method.

b) Cramer's Rule.

VIDEOS

REFERENCES:

NO

UNIT 23

Properties of Angles

SLO'S:

- Define adjacent, complementary, and supplementary angles.
- Define vertically –opposite angles.
- Calculate the followings:

  a) Adjacent angles.

  b) Complementary angle.

  c) Supplementary angle.

  d) Vertically Opposite angles.

  - Calculate unknown angle of a triangle.

VIDEOS

REFERENCES:

STBB general Math 9 & 10 Chapter 10

STBB Math 9 & 10 Chapter 12

PTBB general math class 10 chapter 7

UNIT 24

Parallel Lines

SLO'S:

- Define parallel line.
- Demonstrate through figures the following
- properties of parallel lines.

a) Two lines which are parallel to the same given line are parallel to each other.

b) If three parallel lines are intersected by two transversals in such a way that two intercepts on one transversal are equal to each other , the two intercepts on the second transversal are also equal.

c) A line through the midpoint of a side of a triangle parallel to another side bisects the third side (an application of above property).

- Draw a transversal to intersect two parallel lines and demonstrate corresponding angles, alternate-interior angles, vertically –opposite angles and interior angles on the same side of transversal.
- Describe the following relation between the pairs of angles when a transversal intersects two parallel lines:

a) Pairs of corresponding angles are equal.

b) Pairs of alternate interior angles are equal.

- Pairs of interior angles on the same side of transversal is supplementary, and demonstrate them through figures.

VIDEOS

REFERENCES:

STBB general Math 9 & 10 Chapter 10

STBB Math 9 & 10 Chapter 12

PTBB general math class 10 chapter 7

UNIT 25

Congruent and Similar Figures

SLO'S:

- Identify congruent and similar figures.
- Recognize the symbol of congruency.
- Apply the properties for two figures to congruent or similar.

VIDEOS

REFERENCES:

STBB general Math 9 & 10 Chapter 10

STBB Math 9 & 10 Chapter 12

PTBB general math class 10 chapter 7

UNIT 26

Congruent Triangles

SLO'S:

- Apply following properties for congruency between two triangles:

  a) SSS $\cong SSS$

  b) SAS $\cong SAS$

  c) ASA $\cong ASA$

  d) RHS

VIDEOS

REFERENCES:

STBB general Math 9 & 10 Chapter 10

STBB Math 9 & 10 Chapter 12

PTBB general math class 10 chapter 7

UNIT 27

Quadrilaterals

SLO'S:

- Demonstrate the following properties of a square:
  a) The four sides of a square are equal.
  b) The four angles of a square are right angles.
  c) Diagonals of a square bisect each other and are equal.
- Demonstrate the following properties of a parallelogram:
  a) Opposite side of a parallelogram are equal.
  b) Opposite angles of a parallelogram are equal.
  c) Diagonals of a parallelogram bisect each other.

VIDEOS

REFERENCES:

STBB Math 9 & 10 Chapter 12

PTBB general math class 10 chapter 7

UNIT 28

Circle

SLO'S:

- Describe the following:
    a) A circle and its centre.
    b) Radius.
    c) Diameter.
    d) Chord.
    e) Arc.
    f) Major arcs.
    g) Minor arcs.
    h) Semicircle
    i) Segment of the circle.
- Describe the terms:
    a) Sector and secant of a circle.
    b) Concyclic points.
    c) Tangent to a circle.
    d) Concentric circles.

- Demonstrate the following properties:

  a) The angle in a semicircle is a right angle.

  b) The angles in the same segment of a circle are equal.

  c) The central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.

- Apply the above properties in different geometrical figures.

VIDEOS

REFERENCES:

STBB Math 9 & 10 Chapter 13

STBB general Math 9 & 10 Chapter 11

PTBB general math class 10 chapter 7

PTBB Math class 10 chapter 9

STBB Math 9 & 10 Chapter 13

STBB general Math 9 & 10 Chapter 11

UNIT 29

Construction of Quadrilateral

SLO'S:

- Construct a rectangle when:

  a) Two sides are given.

  b) Diagonal and one side are given.

- Construct a square when its diagonal is given.

- Construct a parallelogram when two adjacent sides and the angle included between them is given.

VIDEOS

REFERENCES:

PTBB general math class 10 chapter 8

UNIT 30

Tangents to the Circle

SLO'S:

- Locate the centre of a given circle.
- Draw a circle passing through three given non-collinear points.
- Draw a tangent to a given circle from a point P when P lies:
  - a) On the circumference.
  - b) Outside the circle.
- Draw the followings:
  - a) Direct common tangent or external tangent
  - b) Transverse common tangent or internal tangent to two equal circles.
- Draw the followings:
  - a) Direct common tangent or external tangent.
  - b) Transverse common tangent or internal tangent to two Unequal circles.
- Draw a tangent to:
  - a) Two unequal touching circles.
  - b) Two unequal intersecting circles.

VIDEOS

REFERENCES:

STBB Math 9 & 10 Chapter 14

STBB general Math 9 & 10 Chapter 11

PTBB math class 10 chapter 13

UNIT 31

Pythagoras Theorem

SLO'S:

i) State Pythagoras theorem.

ii) Solve right angle triangle by using Pythagoras theorem.

What is the Pythagorean Theorem?

You can learn all about the Pythagorean Theorem, but here is a quick summary:

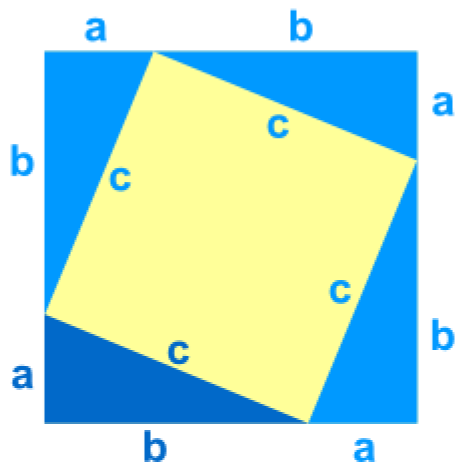
The Pythagorean Theorem says that, *in a right triangle*, the square of a (a^2) plus the square of b (b^2) is equal to the square of c (c^2):

$$a^2 + b^2 = c^2$$

Proof of the Pythagorean Theorem using Algebra

We can show that $\mathbf{a^2 + b^2 = c^2}$ using Algebra

Take a look at this diagram ... it has that "abc" triangle in it (four of them actually):



Area of Whole Square

It is a big square, with each side having a length of **a+b**, so the **total area** is:

$$A = (a+b)(a+b)$$

Area of The Pieces

Now let's add up the areas of all the smaller pieces:

First, the smaller (tilted) square has an area of: c^2

Each of the four triangles has an area of: *ab***2**

So all four of them together is: *4ab***2** = 2ab

Adding up the tilted square and the 4 triangles gives: $A = c^2 + 2ab$

Both Areas Must Be Equal

The area of the **large square** is equal to the area of the **tilted square and the 4 triangles**. This can be written as:

$$(a+b)(a+b) = c^2 + 2ab$$

NOW, let us rearrange this to see if we can get the pythagoras theorem:

Start with: $(a+b)(a+b) = c^2 + 2ab$

Expand (a+b)(a+b): $a^2 + 2ab + b^2 = c^2 + 2ab$

Subtract "2ab" from both sides: $a^2 + b^2 = c^2$

DONE!

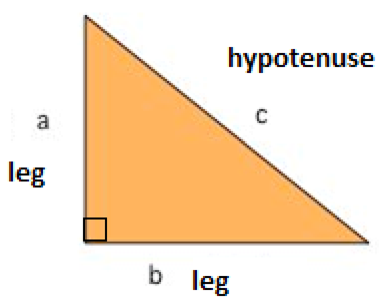
Now we can see why the Pythagorean Theorem works ... and it is actually a **proof** of the Pythagorean Theorem.

This proof came from China over 2000 years ago!

There are many more proofs of the Pythagorean theorem, but this one works nicely.

The Pythagorean Theorem

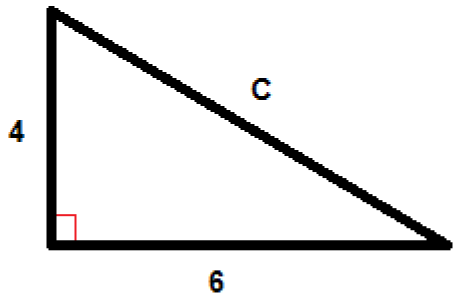
One of the best known mathematical formulas is Pythagorean Theorem, which provides us with the relationship between the sides in a right triangle. A right triangle consists of two legs and a hypotenuse. The two legs meet at a 90° angle and the hypotenuse is the longest side of the right triangle and is the side opposite the right angle.



The Pythagorean Theorem tells us that the relationship in every right triangle is:

a2+b2=c2a2+b2=c2

Example



$$C^2=6^2+4^2$$

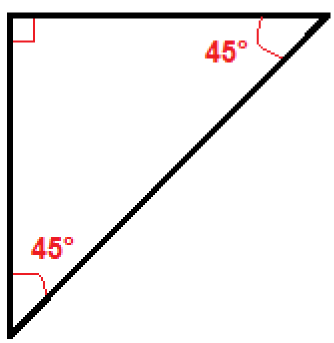
$$C^2=36+16$$

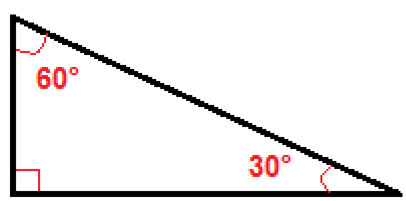
$$C^2=52$$

$$C=\sqrt{52}$$

$$C\approx 7.2$$

There are a couple of special types of right triangles, like the 45°-45° right triangles and the 30°-60° right triangle.

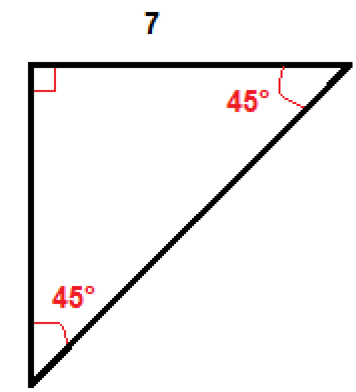




Because of their angles it is easier to find the hypotenuse or the legs in these right triangles than in all other right triangles.

In a 45°-45° right triangle we only need to multiply one leg by $\sqrt{2}$ to get the length of the hypotenuse.

Example



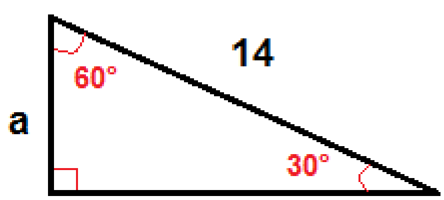
We multiply the length of the leg which is 7 inches by √2 to get the length of the hypotenuse.

$$7 \cdot 2 – \surd \approx 9.97 \cdot 2 \approx 9.9$$

In a 30°-60° right triangle we can find the length of the leg that is opposite the 30° angle by using this formula:

$$a=12 \cdot ca=12 \cdot c$$

Example



To find a, we use the formula above.

$$a=12 \cdot 14a=12 \cdot 14$$

$$a=7$$

The Pythagorean Theorem

Learning Objective(s)

- Use the Pythagorean Theorem to find the unknown side of a right triangle.
- Solve application problems involving the Pythagorean Theorem.

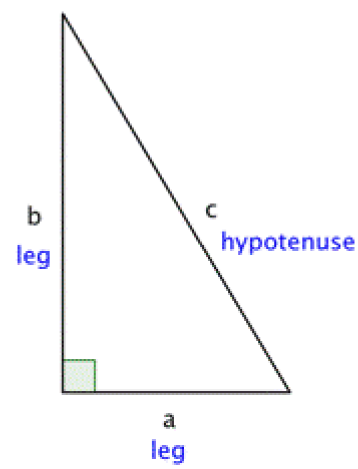
Introduction

A long time ago, a Greek mathematician named Pythagoras discovered an interesting property about right triangles: the sum of the squares of the lengths of each of the triangle's legs is the same as the square of the length of the triangle's hypotenuse. This property—which has many applications in science, art, engineering, and architecture—is now called the Pythagorean Theorem.

Let's take a look at how this theorem can help you learn more about the construction of triangles. And the best part—you don't even have to speak Greek to apply Pythagoras' discovery.

The Pythagorean Theorem

Pythagoras studied right triangles, and the relationships between the legs and the hypotenuse of a right triangle, before deriving his theory.



The Pythagorean Theorem

If *a* and *b* are the lengths of the legs of a right triangle and *c* is the length of the hypotenuse, then the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

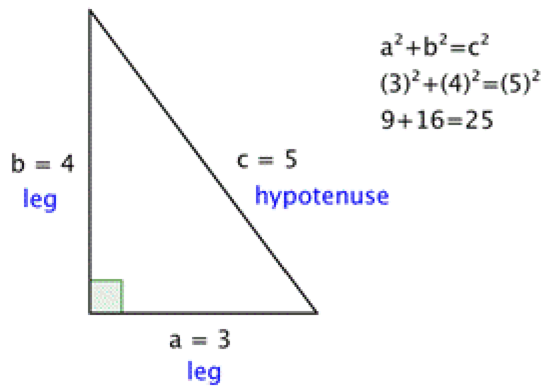
This relationship is represented by the formula: $a^2 + b^2 = c^2$

In the box above, you may have noticed the word "square," as well as the small 2s to the top right of the letters in $a^2 + b^2 = c^2$. To **square** a number means to multiply it by itself. So, for example, to square the number 5 you multiply 5 • 5, and to square the number 12, you multiply 12 • 12. Some common squares are shown in the table below.

Number	**Number Times Itself**	**Square**
1	$1^2 = 1 \cdot 1$	1
2	$2^2 = 2 \cdot 2$	4
3	$3^2 = 3 \cdot 3$	9
4	$4^2 = 4 \cdot 4$	16
5	$5^2 = 5 \cdot 5$	25
10	$10^2 = 10 \cdot 10$	100

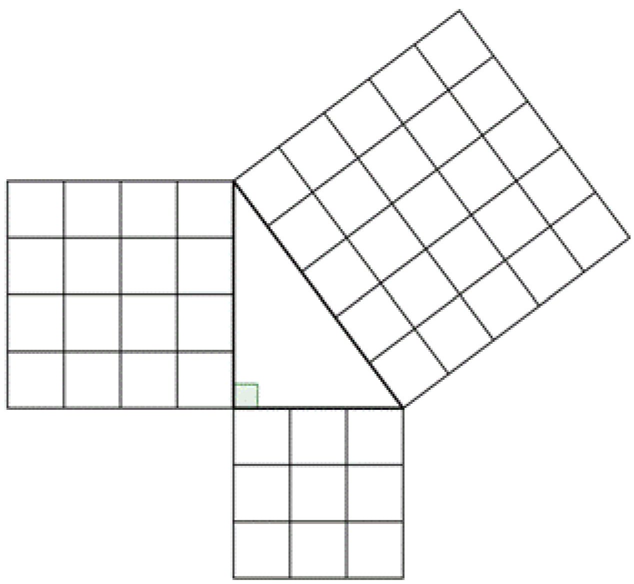
When you see the equation $a^2 + b^2 = c^2$, you can think of this as "the length of side *a* times itself, plus the length of side *b* times itself is the same as the length of side *c* times itself."

Let's try out all of the Pythagorean Theorem with an actual right triangle.



This theorem holds true for this right triangle—the sum of the squares of the lengths of both legs is the same as the square of the length of the hypotenuse. And, in fact, it holds true for all right triangles.

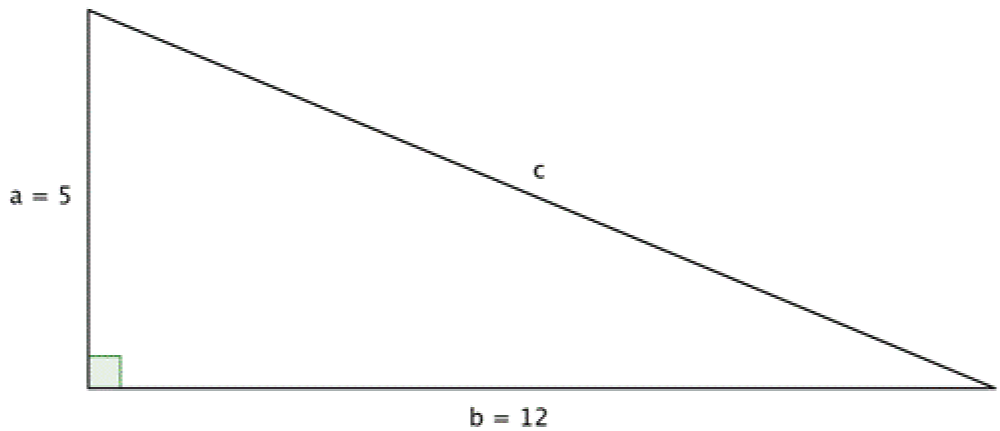
The Pythagorean Theorem can also be represented in terms of area. In any right triangle, the area of the square drawn from the hypotenuse is equal to the sum of the areas of the squares that are drawn from the two legs. You can see this illustrated below in the same 3-4-5 right triangle.



Note that the Pythagorean Theorem only works with *right* triangles.

Finding the Length of the Hypotenuse

You can use the Pythagorean Theorem to find the length of the hypotenuse of a right triangle if you know the length of the triangle's other two sides, called the legs. Put another way, if you know the lengths of *a* and *b*, you can find *c*.



In the triangle above, you are given measures for legs *a* and *b*: 5 and 12, respectively. You can use the Pythagorean Theorem to find a value for the length of *c*, the hypotenuse.

$a^2 + b^2 = c^2$	The Pythagorean Theorem.
$(5)^2 + (12)^2 = c^2$	Substitute known values for *a* and *b*.
$25 + 144 = c^2$	Evaluate.
$169 = c^2$	Simplify. To find the value of *c*, think about a number that, when multiplied by itself, equals 169. Does 10 work? How about 11? 12? 13? (You can use a calculator to multiply if the numbers are unfamiliar.)
$13 = c$	The square root of 169 is 13.

Using the formula, you find that the length of *c*, the hypotenuse, is 13.

In this case, you did not know the value of *c*—you were given the square of the length of the hypotenuse, and had to figure it out from there. When you are given an equation like $169 = c^2$ and are asked to find the value of *c*, this is called finding the **square root** of a number. (Notice you found a number, *c*, whose square was 169.)

Finding a square root takes some practice, but it also takes knowledge of multiplication, division, and a little bit of trial and error. Look at the table below.

Number *x*	**Number *y* which, when multiplied by itself, equals number *x***	**Square root *y***
1	1 • 1	1
4	2 • 2	2
9	3 • 3	3
16	4 • 4	4
25	5 • 5	5
100	10 • 10	10

It is a good habit to become familiar with the squares of the numbers from 0–10, as these arise frequently in mathematics. If you can remember those square numbers—or if you can use a calculator to find them—then finding many common square roots will be just a matter of recall.