



# ZIAUDDIN UNIVERSITY

## EXAMINATION BOARD

### Mathematics

### XI

### Student Resource



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## MATRICES:

### Bigger Matrices

The inverse of a 2x2 is **easy** ... compared to larger matrices (such as a 3x3, 4x4, etc).

For those larger matrices there are three main methods to work out the inverse:

- Inverse of a Matrix using Elementary Row Operations (Gauss-Jordan)
- Inverse of a Matrix using Minors, Cofactors and Adjugate
- Use a computer (such as the Matrix Calculator)

### Conclusion

- The inverse of A is  $A^{-1}$  only when  $A \times A^{-1} = A^{-1} \times A = I$
- To find the inverse of a 2x2 matrix: **swap** the positions of a and d, put **negatives** in front of b and c, and **divide** everything by the determinant (ad-bc).
- Sometimes there is no inverse at all

Topic 1.6: Solution of simultaneous Linear Equation

SLOs:

- Solve a system of two linear equations, using Matrix inversion method and Cramer's rule

Matrices - solving two simultaneous equations sigma-matrices8-2009-1 One of the most important applications of matrices is to the solution of linear simultaneous equations. On this leaflet we explain how this can be done. Writing simultaneous equations in matrix form Consider the simultaneous equations  $x + 2y = 4$   $3x - 5y = 1$  Provided you understand how matrices are multiplied together you will realise that these can be written in matrix form as  $\begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$  Writing  $A = \begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}$ ,  $X = \begin{pmatrix} x \\ y \end{pmatrix}$ , and  $B = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$  we have  $AX = B$  This is the matrix form of the simultaneous equations. Here the only unknown is the matrix X, since A and B are already known. A is called the matrix of coefficients. Solving the simultaneous equations Given  $AX = B$  we can multiply both sides by the inverse of A, provided this exists, to give  $A^{-1}AX = A^{-1}B$  But  $A^{-1}A = I$ , the identity matrix. Furthermore,  $IX = X$ , because multiplying any matrix by an identity matrix of the appropriate size leaves the matrix unaltered. So  $X = A^{-1}B$  if  $AX = B$ , then  $X = A^{-1}B$  This result gives us a method for solving simultaneous equations. All we need do is write them in matrix form, calculate the inverse of the matrix of coefficients, and finally perform a matrix multiplication. www.mathcentre.ac.uk 1 c mathcentre 2009 Example. Solve the simultaneous equations  $x + 2y = 4$   $3x - 5y = 1$  Solution. We have already seen these equations in matrix form:  $\begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ . We need to calculate the inverse of  $A = \begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}$ .  $A^{-1} = \frac{1}{(1)(-5) - (2)(3)} \begin{pmatrix} -5 & -2 \\ -3 & 1 \end{pmatrix} = \frac{1}{-11} \begin{pmatrix} -5 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} 5/11 & 2/11 \\ 3/11 & -1/11 \end{pmatrix}$  Then X is given by  $X = A^{-1}B = \begin{pmatrix} 5/11 & 2/11 \\ 3/11 & -1/11 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 20/11 + 2/11 \\ 12/11 - 1/11 \end{pmatrix} = \begin{pmatrix} 22/11 \\ 11/11 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$  Hence  $x = 2$ ,  $y = 1$  is the solution of the simultaneous equations. Example. Solve the simultaneous equations  $2x + 4y = 2$   $-3x + y = 11$  Solution. In matrix form:  $\begin{pmatrix} 2 & 4 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 11 \end{pmatrix}$ . We need to calculate the inverse of  $A = \begin{pmatrix} 2 & 4 \\ -3 & 1 \end{pmatrix}$ .  $A^{-1} = \frac{1}{(2)(1) - (4)(-3)} \begin{pmatrix} 1 & -4 \\ 3 & 2 \end{pmatrix} = \frac{1}{14} \begin{pmatrix} 1 & -4 \\ 3 & 2 \end{pmatrix}$

$1(2)(1) - (4)(-3) = 1(4) - 3(2) = 4 - 6 = -2$  Then X is given by  $X = A^{-1}B = \frac{1}{14} \begin{pmatrix} 1 & -4 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 11 \end{pmatrix} = \frac{1}{14} \begin{pmatrix} 2 - 44 \\ -28 + 22 \end{pmatrix} = \frac{1}{14} \begin{pmatrix} -42 \\ -6 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$  Hence  $x = -3, y = -2$  is the solution of the simultaneous equations. You should check the solution by substituting  $x = -3$  and  $y = -2$  into both given equations, and verifying in each case that the left-hand side is equal to the right-hand side.

## LOGARITHM:

### How to Do it

To figure out the power of 10, think "how many places do I move the decimal point?"



When the number is 10 or greater, the decimal point has to move **to the left**, and the power of 10 is **positive**.



When the number is smaller than 1, the decimal point has to move **to the right**, so the power of 10 is **negative**.

**Example: 0.0055 is written  $5.5 \times 10^{-3}$**

Because  $0.0055 = 5.5 \times 0.001 = 5.5 \times 10^{-3}$

**Example: 3.2 is written  $3.2 \times 10^0$**

We didn't have to move the decimal point at all, so the power is  $10^0$

But it is now in Scientific Notation

### Check!

After putting the number in Scientific Notation, just check that:

- The "digits" part is between 1 and 10 (it can be 1, but never 10)
- The "power" part shows exactly how many places to move the decimal point

## Why Use It?

Because it makes it easier when dealing with very big or very small numbers, which are common in Scientific and Engineering work.

Example: it is easier to write (and read)  $1.3 \times 10^{-9}$  than 0.0000000013

It can also make calculations easier, as in this example:

**Example: a tiny space inside a computer chip has been measured to be 0.00000256m wide, 0.00000014m long and 0.000275m high.**

**What is its volume?**

Let's first convert the three lengths into scientific notation:

- width:  $0.000\ 002\ 56\text{m} = 2.56 \times 10^{-6}$
- length:  $0.000\ 000\ 14\text{m} = 1.4 \times 10^{-7}$
- height:  $0.000\ 275\text{m} = 2.75 \times 10^{-4}$

Then multiply the digits together (ignoring the  $\times 10$ s):

$$2.56 \times 1.4 \times 2.75 = 9.856$$

Last, multiply the  $\times 10$ s:

$$10^{-6} \times 10^{-7} \times 10^{-4} = 10^{-17} \text{ (easier than it looks, just **add -6, -4 and -7** together)}$$

The result is  **$9.856 \times 10^{-17} \text{ m}^3$**

It is used a lot in Science:

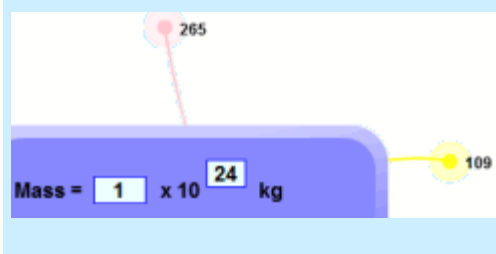
## Example: Suns, Moons and Planets

The Sun has a Mass of  $1.988 \times 10^{30}$  kg.

Easier than writing 1,988,000,000,000,000,000,000,000,000 kg  
(and that number gives a false sense of many digits of accuracy.)

### Play With It!

Use Scientific Notation  
in [Gravity Freeplay](#)



It can also save space! Here is what happens when you double on each square of a chess board:

Values are rounded off, so 53,6870,912 is shown as just  $5 \times 10^8$

That last value, shown as  $9 \times 10^{18}$  is actually 9,223,372,036,854,775,808

### Engineering Notation

Engineering Notation is like Scientific Notation, except that we only use powers of ten that are multiples of 3 (such as  $10^3$ ,  $10^{-3}$ ,  $10^{12}$  etc).

#### Examples:

- 2,700 is written  $2.7 \times 10^3$
- 27,000 is written  $27 \times 10^3$
- 270,000 is written  $270 \times 10^3$
- 2,700,000 is written  $2.7 \times 10^6$

**Example: 0.00012 is written  $120 \times 10^{-6}$**

Notice that the "digits" part can now be between 1 and 1,000 (it can be 1, but never 1,000).

The advantage is that we can replace the  $\times 10$ s with [Metric Numbers](#). So we can use standard words (such as thousand or million), prefixes (such as kilo, mega) or the symbol (k, M, etc)

Example: 19,300 meters is written  $19.3 \times 10^3$  m, or 19.3 km

Example: 0.00012 seconds is written  $120 \times 10^{-6}$  s, or 120 microseconds