ZIAUDDIN UNIVERSITY
EXAMINATION BOARD

Math
X
Teacher Resource

Contents

UNIT 1

TOPIC: Algebraic Expression

SLO'S:

· Define that a rational expression behaves like a rational number.

· Describe a rational expression as a quotient of two polynomials *p(x)* and *q(x),* where *q(x),* is not the zero polynomial.

· Examine whether a given algebraic expression is a

a) Polynomial or not,

b) Rational expression or not.

· Define as a rational expression in its lowest term, if p(x) and q(x) are polynomials with integral coefficients and having no common factor.

· Examine whether a given rational algebraic expression is in its lowest form or not.

· Reduce a given rational expression to its lowest form.

UNIT-1

Section 1-6 : Rational Expressions

We now need to look at rational expressions. A **rational expression** is nothing more than a fraction in which the numerator and/or the denominator are polynomials. Here are some examples of rational expressions.

$$\mathbf{\underline{6 \quad z^2-1m^4+18m+14x^2+6x-10}}$$
$$\mathbf{x\text{-}1 \qquad z^2+5 \qquad m^2-m-6 \qquad 1}$$

The last one may look a little strange since it is more commonly written $4x^2+6x-10$. However, it's important to note that polynomials can be thought of as rational expressions if we need to, although they rarely are.

There is an unspoken rule when dealing with rational expressions that we now need to address. When dealing with numbers we know that division by zero is not allowed. Well the same is true for rational expressions. So, when dealing with rational expressions we will always assume that whatever xx is it won't give division by zero. We rarely write these restrictions down, but we will always need to keep them in mind.

For the first one listed we need to avoid x=1. The second rational expression is never zero in the denominator and so we don't need to worry about any restrictions. Note as well that the numerator of the second rational expression will be zero. That is okay, we just need to avoid division by zero. For the third rational expression we will need to avoid $m=3$ and $m=-2$. The final rational expression listed above will never be zero in the denominator so again we don't need to have any restrictions.

The first topic that we need to discuss here is reducing a rational expression to lowest terms. A rational expression has been **reduced to lowest terms** if all common factors from the numerator and denominator have been canceled. We already know how to do this with number fractions so let's take a quick look at an example.

not reduced to lowest terms ⇒ $\frac{12}{8} = \frac{(4)(3)}{(4)(2)} = \frac{3}{2}$ ⇐ reduced to lowest terms

With rational expression it works exactly the same way.

not reduced to lowest terms ⇒ $\frac{(x+3)(x-1)}{x\,(x+3)} = \frac{x-1}{x}$ ⇐ reduced to lowest term

We do have to be careful with canceling however. There are some common mistakes that students often make with these problems. Recall that in order to cancel a factor it must multiply the whole numerator and the whole denominator. So, the $x+3$ above could cancel since it multiplied the whole numerator and the whole denominator. However, the xx's in the reduced form can't cancel since the xx in the numerator is not times the whole numerator.

To see why the xx's don't cancel in the reduced form above put a number in and see what happens. Let's plug in x=4x=4.

4−14=344−14=−14−14=344−14=−1

Clearly the two aren't the same number!

So, be careful with canceling. As a general rule of thumb remember that you can't cancel something if it's got a "+" or a "-" on one side of it. There is one exception to this rule of thumb with "-" that we'll deal with in an example later on down the road.

Let's take a look at a couple of examples.

Example 1 Reduce the following rational expression to lowest terms.

x2−2x−8x2−9x+20x2−2x−8x2−9x+20
x2−255x−x2x2−255x−x2

x7+2x6+x5x3(x+1)8x7+2x6+x5x3(x+1)8

Discussion

When reducing a rational expression to lowest terms the first thing that we will do is factor both the numerator and denominator as much as possible. That should always be the first step in these problems.

Also, the factoring in this section, and all successive section for that matter, will be done without explanation. It will be assumed that you are capable of doing and/or checking the factoring on your own. In other words, make sure that you can factor!

a) x2−2x−8x2−9x+20x2−2x−8x2−9x+20

Solution

We'll first factor things out as completely as possible. Remember that we can't cancel anything at this point in time since every term has a "+" or a "-" on one side of it! We've got to factor first!

x2−2x−8x2−9x+20=(x−4)(x+2)(x−5)(x−4)x2−2x−8x2−9x+20=(x−4)(x+2)(x−5)(x−4)

At this point we can see that we've got a common factor in both the numerator and the denominator and so we can cancel the xx-4 from both. Doing this gives,

x2−2x−8x2−9x+20=x+2x−5x2−2x−8x2−9x+20=x+2x−5

This is also all the farther that we can go. Nothing else will cancel and so we have reduced this expression to lowest terms.

b) x2−255x−x2x2−255x−x2

Solution

Again, the first thing that we'll do here is factor the numerator and denominator.

x2−255x−x2=(x−5)(x+5)x(5−x)x2−255x−x2=(x−5)(x+5)x(5−x)

At first glance it looks there is nothing that will cancel. Notice however that there is a term in the denominator that is almost the same as a term in the numerator except all the signs are the opposite.

We can use the following fact on the second term in the denominator.

a−b=−(b−a)OR−a+b=−(a−b)a−b=−(b−a)OR−a+b=−(a−b)

This is commonly referred to as **factoring a minus sign out** because that is exactly what we've done. There are two forms here that cover both possibilities that we are liable to run into. In our case however we need the first form.

Because of some notation issues let's just work with the denominator for a while.

x(5−x)=x[−(x−5)]=x[(−1)(x−5)]=x(−1)(x−5)=(−1)(x)(x−5)=−x(x−5)x(5−x)=x[−(x−5)]=x[(−1)(x −5)]=x(−1)(x−5)=(−1)(x)(x−5)=−x(x−5)

Notice the steps used here. In the first step we factored out the minus sign, but we are still multiplying the terms and so we put in an added set of brackets to make sure that we didn't forget that. In the second step we acknowledged that a minus sign in front is the same as multiplication by "-1". Once we did them in the third step. Next, we recalled that we change the order of a multiplication if we need to so we flipped the xx and the "-1". Finally, we dropped the "-1" and just went back to a negative sign in the front.

Typically, when we factor out minus signs we skip all the intermediate steps and go straight to the final step.

Let's now get back to the problem. The rational expression becomes,

x2−255x−x2=(x−5)(x+5)−x(x−5)x2−255x−x2=(x−5)(x+5)−x(x−5)

At this point we can see that we do have a common factor and so we can cancel the *x-5*.

x2−255x−x2=x+5−x=−x+5xx2−255x−x2=x+5−x=−x+5x

c) x7+2x6+x5x3(x+1)8x7+2x6+x5x3(x+1)8

Solution

In this case the denominator is already factored for us to make our life easier. All we need to do is factor the numerator.

x7+2x6+x5x3(x+1)8=x5(x2+2x+1)x3(x+1)8=x5(x+1)2x3(x+1)8x7+2x6+x5x3(x+1)8=x5(x2+2x +1)x3(x+1)8=x5(x+1)2x3(x+1)8

Now we reach the point of this part of the example. There are 5 xx's in the numerator and 3 in the denominator so when we cancel there will be 2 left in

denominator so when we cancel there will be 6 left in the denominator. Here is the rational expression reduced to lowest terms.

x7+2x6+x5x3(x+1)8=x2(x+1)6x7+2x6+x5x3(x+1)8=x2(x+1)6

Before moving on let’s briefly discuss the answer in the second part of this example. Notice that we moved the minus sign from the denominator to the front of the rational expression in the final form. This can always be done when we need to. Recall that the following are all equivalent.

−ab=−ab=a−b−ab=−ab=a−b

In other words, a minus sign in front of a rational expression can be moved onto the whole numerator or whole denominator if it is convenient to do that. We do have to be careful with this however. Consider the following rational expression.

−x+3x+1−x+3x+1

In this case the “-” on the xx can’t be moved to the front of the rational expression since it is only on the xx. In order to move a minus sign to the front denominator. So, if we factor a minus out of the numerator we could then move it into the front of the rational expression as follows,

−x+3x+1=−(x−3)x+1=−x−3x+1−x+3x+1=−(x−3)x+1=−x−3x+1

The moral here is that we need to be careful with moving minus signs around in rational expressions.

We now need to move into adding, subtracting, multiplying and dividing rational expressions.

Let’s start with multiplying and dividing rational expressions. The general formulas are as following

ab·cd=acbd =ab÷cd=ab·dc

Note the two different forms for denoting division. We will use either as needed so make sure you are familiar with both. Note as well that to do division of rational expressions all that we need to do is multiply the numerator by the reciprocal of the denominator (*i.e.* the fraction with the numerator and denominator switched).

Before doing a couple of examples there are a couple of *special* cases of division that we should look at. In the general case above both the numerator and the denominator of the rational expression are fractions, however, what if one of them isn’t a fraction. So let’s look at the following cases.

acdabc

Students often make mistakes with these initially. To correctly deal with these we will turn the numerator (first case) or denominator (second case) into a fraction and then do the general division on them.

acd=a1cd=a1·dc=adcabc=abc1=ab·1c=abc

Be careful with these cases. It is easy to make a mistake with these and incorrectly do the division.

Now let’s take a look at a couple of examples.

Example 2 Perform the indicated operation and reduce the answer to lowest terms.

x2−5x−14x2−3x+2·x2−4x2−14x+49x2−5x−14x2−3x+2·x2−4x2−14x+49
m2−9m2+5m+6÷3−mm+2m2−9m2+5m+6÷3−mm+2
y2+5y+4y2−1y+5y2+5y+4y2−1y+5

Solutions

Notice that with this problem we have started to move away from xx as the main variable in the examples. Do not get so used to seeing xx’s that you always expect them. The problems will work the same way regardless of the letter we use for the variable so don’t get excited about the different letters here.

a) x2−5x−14x2−3x+2·x2−4x2−14x+49x2−5x−14x2−3x+2·x2−4x2−14x+49

Solution

Okay, this is a multiplication. The first thing that we should always do in the multiplication is to factor everything in sight as much as possible.

x2−5x−14x2−3x+2·x2−4x2−14x+49=(x−7)(x+2)(x−2)(x−1)·(x−2)(x+2)(x−7)2x2−5x−14x2−3x+2·x2−4x2−14x+49=(x−7)(x+2)(x−2)(x−1)·(x−2)(x+2)(x−7)2

Now, recall that we can cancel things across a multiplication as follows.

abk·ckd=ab·cdabk·ckd=ab·cd

Note that this ONLY works for multiplication and NOT for division!

In this case we do have multiplication so cancel as much as we can and then do the multiplication to get the answer.

x2−5x−14x2−3x+2·x2−4x2−14x+49=(x+2)(x−1)·(x+2)(x−7)=(x+2)2(x−1)(x−7)x2−5x−14x2−3x+2·x2−4x2−14x+49=(x+2)(x−1)·(x+2)(x−7)=(x+2)2(x−1)(x−7)

b) m2−9m2+5m+6÷3−mm+2m2−9m2+5m+6÷3−mm+2

Solution

With division problems it is very easy to mistakenly cancel something that shouldn't be canceled and so the first thing we do here (before factoring!!!!) is do the division. Once we've done the division we have a multiplication problem and we factor as much as possible, cancel everything that can be canceled and finally do the multiplication.

So, let's get started on this problem.

m2−9m2+5m+6÷3−mm+2=m2−9m2+5m+6·m+23−m=(m−3)(m+3)(m+3)(m+2)·(m+2)(3−m)m2−9m2+5m+6÷3−mm+2=m2−9m2+5m+6·m+23−m=(m−3)(m+3)(m+3)(m+2)·(m+2)(3−m)

Now, notice that there will be a lot of canceling here. Also notice that if we factor a minus sign out of the denominator of the second rational expression. Let's do some of the canceling and then do the multiplication.

m2−9m2+5m+6÷3−mm+2=(m−3)1·1−(m−3)=(m−3)−(m−3)m2−9m2+5m+6÷3−mm+2=(m−3)1·1−(m−3)=(m−3)−(m−3)

Remember that when we cancel all the terms out of a numerator or denominator there is actually a "1" left over! Now, we didn't finish the canceling to make a point. Recall that at the start of this discussion we said that as a rule of thumb we can only cancel terms if there isn't a "+" or a "-" on either side of it with one exception for the "-". We are now at that exception. If there is a "-" in front of the whole numerator or denominator, as we've got here, then we can still cancel the term. In this case the "-" acts as a "-1" that is multiplied by the whole denominator and so is a factor instead of an addition or subtraction. Here is the final answer for this part.

m2−9m2+5m+6÷3−mm+2=1−1=−1m2−9m2+5m+6÷3−mm+2=1−1=−1

In this case all the terms canceled out and we were left with a number. This doesn't happen all that often, but as this example has shown it clearly can happen every once in a while so don't get excited about it when it does happen.

c) y2+5y+4y2−1y+5y2+5y+4y2−1y+5

Solution

This is one of the special cases for division. So, as with the previous part, we will first do the division and then we will factor and cancel as much as we can.

Here is the work for this part.

y2+5y+4y2−1y+5=(y2+5y+4)(y+5)y2−1=(y+1)(y+4)(y+5)(y+1)(y−1)=(y+4)(y+5)y−1y2+5y+4y2−1y+5=(y2+5y+4)(y+5)y2−1=(y+1)(y+4)(y+5)(y+1)(y−1)=(y+4)(y+5)y−1

Okay, it's time to move on to addition and subtraction of rational expressions. Here are the general formulas.

ac+bc=a+bcac−bc=a−bcac+bc=a+bcac−bc=a−bc

As these have shown we've got to remember that in order to add or subtract rational expression or fractions we MUST have common denominators. If we don't have common denominators then we need to first get common denominators.

Let's remember how do to do this with a quick number example.

56−3456−34

In this case we need a common denominator and recall that it's usually best to use the **least common denominator**, often denoted **lcd**. In this case the least common denominator is 12. So we need to get the denominators of these two fractions to a 12. This is easy to do. In the first case we need to multiply the denominator by 2 to get 12 so we will multiply the numerator and denominator of the first fraction by 2. Remember that we've got to multiply both the numerator and denominator by the same number since we aren't allowed to actually change the problem and this is equivalent to multiplying the fraction by 1 since aa=1aa=1. For the second term we'll need to multiply the numerator and denominator by a 3.

56−34=5(2)6(2)−3(3)4(3)=1012−912=10−912=11256−34=5(2)6(2)−3(3)4(3)=1012−912=10−912=112

Now, the process for rational expressions is identical. The main difficulty is in finding the least common denominator. However, there is a really simple process for finding the least common denominator for rational expressions. Here is it.

1. Factor all the denominators.
2. Write down each factor that appears at least once in any of the denominators. Do NOT write down the power that is on each factor, only write down the factor
3. Now, for each factor written down in the previous step write down the largest power that occurs in all the denominators containing that factor.
4. The product all the factors from the previous step is the least common denominator.

Let's work some examples.

Example 3 Perform the indicated operation.

46x2−13x5+52x346x2−13x5+52x3
2z+1−z−1z+22z+1−z−1z+2
yy2−2y+1−2y−1+3y+2yy2−2y+1−2y−1+3y+2
2xx2−9−1x+3−2x−32xx2−9−1x+3−2x−3
4y+2−1y+14y+2−1y+1

a) 46x2−13x5+52x346x2−13x5+52x3

Solution

For this problem there are coefficients on each term in the denominator so we'll first need the least common denominator for the coefficients. This is 6. Now, xx (by itself with a power of 1) is the only factor that occurs in any of the denominators. So, the least common denominator for this part is xx with the largest power that occurs on all the xx's in the problem, which is 5. So, the least common denominator for this set of rational expression is

lcd : 6x5lcd : 6x5

So, we simply need to multiply each term by an appropriate quantity to get this in the denominator and then do the addition and subtraction. Let's do that.

46x2−13x5+52x3=4(x3)6x2(x3)−1(2)3x5(2)+5(3x2)2x3(3x2)=4x36x5−26x5+15x26x5=4x3−2+15x26x546x2−13x5+52x3=4(x3)6x2(x3)−1(2)3x5(2)+5(3x2)2x3(3x2)=4x36x5−26x5+15x26x5=4x3−2+15x26x5

b) 2z+1−z−1z+22z+1−z−1z+2

Solution

In this case there are only two factors and they both occur to the first power and so the least common denominator is.

lcd : (z+1)(z+2)lcd : (z+1)(z+2)

Now, in determining what to multiply each part by simply compare the current denominator to the least common denominator and multiply top and bottom by whatever is "missing". In the first term we're "missing" a z+2z+2 and so that's what we multiply the numerator and denominator by. In the second term we're "missing" a z+1z+1 and so that's what we'll multiply in that term.

Here is the work for this problem.

2z+1−z−1z+2=2(z+2)(z+1)(z+2)−(z−1)(z+1)(z+2)(z+1)=2(z+2)−(z−1)(z+1)(z+1)(z+2)2z+1−z−1z+2=2(z+2)(z+1)(z+2)−(z−1)(z+1)(z+2)(z+1)=2(z+2)−(z−1)(z+1)(z+1)(z+2)

The final step is to do any multiplication in the numerator and simplify that up as much as possible.

2z+1−z−1z+2=2z+4−(z2−1)(z+1)(z+2)=2z+4−z2+1(z+1)(z+2)=−z2+2z+5(z+1)(z+2)2z+1−z−1z+2=2z+4−(z2−1)(z+1)(z+2)=2z+4−z2+1(z+1)(z+2)=−z2+2z+5(z+1)(z+2)

Be careful with minus signs and parenthesis when doing the subtraction.

Roots and Radicals: (lesson 1 of 3)

Rational Expressions with the Same Denominator

To add/subtract rational expressions with the same denominator

1. Add/subtract the numerators. Write this sum/difference as the numerator over the common denominator.
2. Reduce to lowest terms.

Example 1

Simplify the following:

$$\frac{4x}{5y}+\frac{6x}{5y}$$

Solution

These fractions already have a common denominator

1: Write this sum as the numerator over the common denominator:

$$\frac{4x}{5y}+\frac{6x}{5y}=\frac{4x+6x}{5y}$$

2: Reduce to lowest terms:

$$\frac{4x}{5y}+\frac{6x}{5y}=\frac{4x+6x}{5y}=\frac{10x}{5y}=\frac{2x}{y}$$

Example 2

Simplify the following:

$$\frac{4x-1}{4x+1}-\frac{2x-9}{x+4}$$

Solution

Again, these already have a common denominator

1: Write this sum as the numerator over the common denominator:

$$\frac{4x-1}{4x+1} - \frac{2x-9}{x+4} = \frac{(4x-1)-(2x-9)}{x+4}$$

2: Reduce to lowest terms:

$$\frac{4x-1}{4x+1} - \frac{2x-9}{x+4} = \frac{(4x-1)-(2x-9)}{x+4}$$

$$= \frac{4x-1-2x+9}{x+4} =$$

$$= \frac{2x+8}{x+4} =$$

$$= \frac{2(x+4)}{x+4} = 2$$

Adding or Subtracting Rational Expressions with Different Denominators

1. Factor each denominator completely.
2. Build the LCD of the denominators.
3. Rewrite each rational expression with the LCD as the denominator.
4. Add/subtract the numerators.

Example 3:

Simplify the following:

$5x-1x2-3x+2+32x-4$

Solution 3:

1: Factor each denominator completely.

$5x-1x2-3x+2+22x-4=5x-1(x-1)(x-2)+32(x-2)$

2: Build the LCD of the denominators.

$LCD=2(x-1)(x-2)$

3: Rewrite each rational expression with the LCD as the denominator.

$$5x-1x2-3x+2+32x-4=5x-1(x-1)(x-2)+32(x-2)=$$

$$=2(5x-1)2(x-1)(x-2)+3(x-1)2(x-1)(x-2)$$

4: Add the numerators.

$$5x-1x2-3x+2+32x-4=5x-1(x-1)(x-2)+32(x-2)=$$

$$=2(5x-1)2(x-1)(x-2)+3(x-1)2(x-1)(x-2)=$$

$$=2(5x-1)+3(x-1)2(x-1)(x-2)=$$

$$=13x-52(x-1)(x-2)$$

Example 4:

Simplify the following:

$$\frac{5x+1}{x2-2x-3} - \frac{5x-3}{3x2-x-6}$$

Solution 4:

1: Factor each denominator completely.

$$\frac{5x+1}{x2-2x-3} - \frac{5x-3}{3x2-x-6} = \frac{5x+1}{(x-3)(x+1)} - \frac{5x-3}{(x-3)(x+2)}$$

2: Build the LCD of the denominators.

$$LCD=(x-3)(x+1)(x+2)$$

3: Rewrite each rational expression with the LCD as the denominator.

$$\frac{5x+1}{} - \frac{5x-3}{} = \frac{5x+1}{x2-2x-3} - \frac{5x-3}{x2-x-6} \quad (x-3)(x+1) \quad (x-3)(x+2)$$

$$=\frac{(5x+1)(x+2)}{(x-3)(x+1)(x+2)} - \frac{(5x-3)(x+1)}{(x-3)(x+1)(x+2)}$$

4: Subtract the numerators.

$$\frac{5x+1}{x2-2x-3} - \frac{5x-3}{x2-x-6} = \frac{5x+1}{(x-3)(x+1)} - \frac{5x-3}{(x-3)(x+2)}$$

$$=\frac{(5x+1)(x+2)}{(x-3)(x+1)(x+2)} - \frac{(5x-3)(x+1)}{(x-3)(x+1)(x+2)}$$

$$=\frac{(5x+1)(x+2)-(5x-3)(x+1)=}{(x-3)(x+1)(x+2)}$$

$$=\frac{(5x2+10x+x+2)-(5x2+5x-3x-3)=}{(x-3)(x+1)(x+2)}$$

$$=\frac{5x2+10x+x+2-5x2-5x+3x+3=}{(x-3)(x+1)(x+2)}$$

$$=\frac{9x+5}{(x-3)(x+1)(x+2)}$$

VIDEOS

REFERENCES:

- NSM book 2 6th edition chapter # 3
- PTBB General Math class 10 Chapter 1
- STBB General Math book Class 9-10 Chapter 7

UNIT 2

Algebraic Formulas

SLO'S:

- Calculate the of and ab when the given values are a+b and a-b ,the important formula are the followings:

(a + b) 2 + (a – b) 2 = 2(+)

(a + b) 2 - (a – b) 2 = 4a

- Calculation for formula :

= + + + 2ab + 2bc + 2ca

a) Calculate the value + + when the values are given and ab + bc + ca

b) Calculate the value of a +b + c when the values are given + + and ab + bc + ca

c) Calculate the value of ab + bc + ca when the values are given + + and a +b + c

- Calculate the value of + when the values are given a + b and ab the required formula of the following:

(a+ b) 3=+ (a + b) ()

- Calculate the value of when the values are given a and ab the required formula of the following:

$(a)^3$ =)

- Calculate the continued product by following formula:

(x + y)(x ()(

Purplemath

When you learn to factor quadratics, there are three other formulas that they usually introduce at the same time. The first is the "difference of squares" formula.

Remember from your translation skills that a "difference" means a "subtraction". So a difference of squares is something that looks like $x^2 – 4$. That's because $4 = 2^2$, so we really have $x^2 – 2^2$, which is a difference of squares.

To factor this, I'll start by writing my parentheses, in the same way as usual for factoring:

$x^2 - 4 = (x \quad)(x \quad)$

For this quadratic factorization, I need factors of –4 that add up to zero, so I'll use –2 and +2:

$x^2 - 4 = (x - 2)(x + 2)$

(Review Factoring Quadratics, if the steps in this example didn't make sense to you.)

Note that we had $x^2 - 2^2$, and ended up with $(x - 2)(x + 2)$. Differences of squares (being something squared minus something else squared) always work this way:

For $a^2 - b^2$, I start by doing the parentheses:

$(\quad)(\quad)$

Then I put the first squared thing in front:

$(a \quad)(a \quad)$

...and I put the second squared thing in back:

$(a \quad b)(a \quad b)$

...and then I alternate the signs in the middles:

$(a - b)(a + b)$

Because the factoring always works out exactly the same way, we can turn it into a formula:

Difference-of-Squares Formula:

For a difference of squares $a^2 - b^2$, the factorization is:

$(a - b)(a + b)$

Memorize this formula! It will come in handy later, especially when you get to rational expressions (polynomial fractions). And you'll probably be expected to know this formula for your next test.

By the way, no, the order of the factors doesn't matter. Since multiplication is commutative (that is, since you can move the factors around without changing the value of the product), the difference of squares can also be stated as:

$(a + b)(a - b)$

Don't get hung up on the order of the factors. Either way is fine.

Here are examples of some typical homework problems:

- *Factor* $x^2 - 16$

This quadratic can be restated as $x^2 - 4^2$, which is a difference of squares. Applying the formula, I get:

$x^2 - 16 = x^2 - 4^2$

$= \mathbf{(x - 4)(x + 4)}$

- *Factor* $4x^2 - 25$

This quadratic is $(2x)^2 - 5^2$ so, applying the formula, I get:

$4x^2 - 25 = (2x)^2 - 5^2$

$= \mathbf{(2x - 5)(2x + 5)}$

- *Factor* $9x^6 - y^8$

This can be restated as $(3x^3)^2 - (y^4)^2$, so I get:

$9x^6 - y^8 = (3x^3)^2 - (y^4)^2$

= $\mathbf{(3x^3 - y^4)(3x^3 + y^4)}$

- *Factor* $x^4 - 1$

This is $(x^2)^2 - 1^2$ so, applying the formula, I get:

$x^4 - 1 = (x^2)^2 - 1^2$

$= (x^2 - 1)(x^2 + 1)$

Note that I'm not done yet, because one of the factors I got — namely, the $x^2 - 1$ factor — is itself a difference of squares, so I need to apply the formula again to get the fully-factored form. Since $x^2 - 1 = x^2 - 1^2 = (x - 1)(x + 1)$, then:

$x^4 - 1 = (x^2)^2 - 1^2$

$= (x^2 - 1)(x^2 + 1)$

$= ((x)^2 - (1)^2)(x^2 + 1)$

= $\mathbf{(x - 1)(x + 1)(x^2 + 1)}$

The answer to this last exercise depended on the fact that 1, to any power at all, is still just 1.

Warning: Never forget that this formula is for the *difference* of squares (with variables); the polynomial *sum* of squares is always prime (that is, it can't be factored with whole numbers or fractions).

The other two special factoring formulas you'll need to memorize are very similar to one another; they're the formulas for factoring the sums and the differences of cubes. Here are the two formulas:

Factoring a Sum of Cubes:

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Factoring a Difference of Cubes:

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

You'll learn in more advanced classes how they came up with these formulas. For now, just memorize them.

To help with the memorization, first notice that the *terms* in each of the two factorization formulas are exactly the same. Then notice that each formula has only one "minus" sign. The distinction between the two formulas is in the location of that one "minus" sign:

For the *difference* of cubes, the "minus" sign goes in the linear factor, $a - b$; for the *sum* of cubes, the "minus" sign goes in the quadratic factor, $a^2 - ab + b^2$.

Some people use the mnemonic "**SOAP**" to help keep track of the signs; the letters stand for the linear factor having the "same" sign as the sign in the middle of the original expression, then the quadratic factor starting with the "opposite" sign from what was in the original expression, and finally the second sign inside the quadratic factor is "always positive".

$a^3 \pm b^3$ = (a [**Same** sign] b)(a^2 [**Opposite** sign] ab [**Always Positive**] b^2)

Whatever method best helps you keep these formulas straight, use it, because you should not assume that you'll be given these formulas on the test. You should expect to need to know them.

Note: The quadratic portion of each cube formula *does not factor*, so don't waste time attempting to factor it. Yes, $a^2 - 2ab + b^2$ and $a^2 + 2ab + b^2$ factor, but that's because of the 2's on their middle terms. These sum- and difference-of-cubes formulas' quadratic terms *do not have* that "2", and thus *cannot* factor.

When you're given a pair of cubes to factor, carefully apply the appropriate rule. By "carefully", I mean "using parentheses to keep track of everything, especially the negative signs". Here are some typical problems:

- **Factor $x^3 - 8$**

This is equivalent to $x^3 - 2^3$. With the "minus" sign in the middle, this is a difference of cubes. To do the factoring, I'll be plugging x and 2 into the difference-of-cubes formula. Doing so, I get:

$x^3 - 8 = x^3 - 2^3$

$= (x - 2)(x^2 + 2x + 2^2)$

$=$ **$(x - 2)(x^2 + 2x + 4)$**

- **Factor $27x^3 + 1$**

The first term contains the cube of 3 and the cube of *x*. But what about the second term?

Before panicking about the lack of an apparent cube, I remember that 1 can be regarded as having been raised to any power I like, since 1 to any power is still just 1. In this case the power I'd like is 3, since this will give me a sum of cubes. This means that the expression they've given me can be expressed as:

$(3x)^3 + 1^3$

So, to factor, I'll be plugging $3x$ and 1 into the sum-of-cubes formula. This gives me:

$27x^3 + 1 = (3x)^3 + 1^3$

$= (3x + 1)((3x)^2 - (3x)(1) + 1^2)$

$=$ **$(3x + 1)(9x^2 - 3x + 1)$**

- **Factor $x^3y^6 - 64$**

First, I note that they've given me a binomial (a two-term polynomial) and that the power on the *x* in the first term is 3 so, even if I weren't working in the "sums and differences of cubes" section of my textbook, I'd be on notice that maybe I should be thinking in terms of those formulas.

Looking at the other variable, I note that a power of 6 is the cube of a power of 2, so the other variable in the first term can be expressed in terms of cubing, too; namely, as the cube of the square of *y*.

The second term is 64, which I remember is the cube of 4. (If I didn't remember, or if I hadn't been certain, I'd have grabbed my calculator and tried cubing stuff until I got the right value, or else I'd have taken the cube root of 64.)

So I now know that, with the "minus" in the middle, this is a difference of two cubes; namely, this is:

$(xy^2)^3 - 4^3$

Plugging into the appropriate formula, I get:

$x^3y^6 - 64 = (xy^2)^3 - 4^3$

$= (xy^2 - 4)((xy^2)^2 + (xy^2)(4) + 4^2)$

= $\mathbf{(xy^2 - 4)(x^2y^4 + 4xy^2 + 16)}$

- **Using an appropriate formula, factor $16x^3 - 250$.**

Um... I know that 16 is *not* a cube of anything; it's actually equal to 2^4. What's up?

What's up is that they expect me to use what I've learned about simple factoring to first convert this to a difference of cubes. Yes, $16 = 2^4$, but $8 = 2^3$, a cube. I can get 8 from 16 by dividing by 2. What happens if I divide 250 by 2? I get 125, which is the cube of 5. So what they've given me can be restated as:

$2(2^3x^3 - 5^3)$

I can apply the difference-of-cubes formula to what's inside the parentheses:

$2^3x^3 - 5^3 = (2x)^3 - (5)^3$

$= (2x - 5)((2x)^2 + (2x)(5) + (5)^2)$

$= (2x - 5)(4x^2 + 10x + 25)$

Putting it all together, I get a final factored form of:

$\mathbf{2(2x - 5)(4x^2 + 10x + 25)}$

ying the difference-of-cubes formula, since $125 = 5^3$. But what about that "minus" sign in front?

Since neither of the factoring formulas they've given me includes a "minus" in front, maybe I can factor the "minus" out...?

$-x^3 - 125 = -1x^3 - 125$

$= -1(x^3 + 125)$

Aha! Now what's inside the parentheses is a *sum* of cubes, which I can factor. I've got the sum of the cube of x and the cube of 5, so:

$x^3 + 5^3 = (x + 5)((x)^2 - (x)(5) + (5)^2)$

$= (x + 5)(x^2 - 5x + 25)$

Putting it all together, I get:

$\mathbf{-1(x + 5)(x^2 - 5x + 25)}$

You can use the Mathway widget below to practice factoring a sum of cubes. Try the entered exercise, or type in your own exercise. Then click the button to compare your answer to Mathway's. (Or skip the widget and continue with the lesson.)

There is one "special" factoring type that can actually be done using the usual methods for factoring, but, for whatever reason, many texts and instructors make a big deal of treating this case separately. "Perfect square trinomials" are quadratics which are the results of squaring binomials. (Remember that "trinomial" means "three-term polynomial".) For instance:

$(x + 3)^2$

$= (x + 3)(x + 3)$

$= x^2 + 6x + 9$

...so $x^2 + 6x + 9$ is a perfect square trinomial.

Recognizing the pattern to perfect squares isn't a make-or-break issue — these are quadratics that you can factor in the usual way — but noticing the pattern can be a time-saver occasionally, which can be helpful on timed tests.

The trick to seeing this pattern is really quite simple: If the first and third terms are squares, figure out what they're squares of. Multiply those things, multiply that product by 2, and then compare your result with the original quadratic's middle term. If you've got a match (ignoring the sign), then you've got a perfect-square trinomial. And the original binomial that they'd squared was the sum (or difference) of the square roots of the first and third terms, together with the sign that was on the middle term of the trinomial.

Perfect-square trinomials are of the form:

$a^2x^2 \pm 2axb + b^2$

...and are expressed in squared-binomial form as:

$(ax \pm b)^2$

How does this look, in practice?

- *Is $x^2 + 10x + 25$ a perfect square trinomial? If so, write the trinomial as the square of a binomial.*

Well, the first term, x^2, is the square of x. The third term, 25, is the square of 5. Multiplying these two, I get $5x$.

Multiplying this expression by 2, I get $10x$. This is what I'm needing to match, in order for the quadratic to fit the pattern of a perfect-square trinomial. Looking at the original quadratic they gave me, I see that the middle term is $10x$, which is what I needed. So this is indeed a perfect-square trinomial:

$(x)^2 + 2(x)(5) + (5)^2$

But what was the original binomial that they'd squared?

I know that the first term in the original binomial will be the first square root I found, which was x. The second term will be the second square root I found, which was 5. Looking back at the original quadratic, I see that the sign on the middle term was a "plus". This means that I'll have a "plus" sign between the x and the 5. Then this quadratic is:

a perfect square, with

$\mathbf{x^2 + 10x + 25 = (x + 5)^2}$

- *Write* $16x^2 - 48x + 36$ *as a squared binomial.*

The first term, $16x^2$, is the square of $4x$, and the last term, 36, is the square of 6.

$(4x)^2 - 48x + 6^2$

Actually, since the middle term has a "minus" sign, the 36 will need to be the square of –6 if the pattern is going to work. Just to be sure, I'll make sure that the middle term matches the pattern:

$(4x)(-6)(2) = -48x$

It's a match to the original quadratic they gave me, so that quadratic fits the pattern of being a perfect square:

$(4x)^2 + (2)(4x)(-6) + (-6)^2$

I'll plug the $4x$ and the –6 into the pattern to get the original squared-binomial form:

$16x^2 - 48x + 36 = \mathbf{(4x - 6)^2}$

- *Is* $4x^2 - 25x + 36$ *a perfect square trinomial?*

The first term, $4x^2$, is the square of $2x$, and the last term, 36, is the square of 6 (or, in this case, –6, if this is a perfect square).

According to the pattern for perfect-square trinomials, the middle term must be:

$(2x)(-6)(2) = -24x$

However, looking back at the original quadratic, it had a middle term of $-25x$, and this does not match what the pattern requires. So:

this is *not* a perfect square trinomial.

- *Factor $x^4 - 2x^2 + 1$ fully.*

If I use the regular methods for factoring quadratic-type polynomials, I can factor this just fine. But what if this is in the homework for the section in my textbook on perfect-square binomials? Naturally, I'm going to be thinking that the author is expecting me to notice a perfect square. So:

The first term is x^4, whose square root is x^2. The third term is 1, whose square root is just 1. Does the middle term, $2x^2$, fit the pattern for perfect-square binomials? I'll check:

$2(x^2)(1) = 2x^2$

It's a match to the original polynomial, so this is a perfect-square trinomial. With the "minus" on the middle term of what they gave me, the original squared-binomial form looks like:

$(x^2 - 1)^2$

Hmm... The instructions say to "factor fully". That's often a clue that there may be some more factoring that I could, after the usual bit is completed. Can I factor any more here?

Yes, I can. Looking inside the parentheses, I notice that I have a difference of squares, which I can factor:

$x^2 - 1 = (x - 1)(x + 1)$

Putting the square on everything, I end up with a fully-factoring answer of:

$x^4 - 2x^2 + 1 = (x^2 - 1)^2$

$= ((x - 1)(x + 1))^2$

= $\mathbf{(x - 1)^2(x + 1)^2}$

That's really all there is to perfect squares.

You've learned the difference-of-squares formula and the difference- and sum-of-cubes formulas. But how do you know which formula to use, and when to use it?

First off, to use any of these formulas, you have to have only two terms in your polynomial. If you've factored out everything you can and you're still left with two terms with a square or a cube in them, then you should look at using one of these formulas. For instance, $6x^2 + 6x$ is two terms, but you can factor out a $6x$, giving you $6x^2 + 6x = 6x(x + 1)$. Since the bit inside the parentheses does not have a squared or a cubed variable in it, you can't apply any of these special factoring formulas. And you don't need to, since it's already fully factored — you can't go further than just plain old "x".

On the other hand, $2x^2 – 162 = 2(x^2 – 81)$, and $x^2 – 81$ is a quadratic. When you see that you have a two-term non-linear polynomial, check to see if it fits any of the formulas. In this case, you've got a difference of squares, so apply that formula: $2x^2 – 162 = 2(x^2 – 81) = 2(x – 9)(x + 9)$.

Warning: Always remember that, in cases like $2x^2 + 162$, all you can do is factor out the 2; the *sum* of squares doesn't factor! $2x^2 + 162 = 2(x^2 + 81)$. (Your book may call $x^2 + 81$ "prime", "unfactorable", or "irreducible". These terms all mean the same thing.)

There is one special case for applying these formulas. Take a look at $x^6 – 64$. Is this expression a difference of squares, being ($(x^3)^2 – 8^2$), or a difference of cubes, being ($(x^2)^3 – 4^3$)? Actually, it's both. You can factor this difference in either of two ways:

factoring a difference of squares, followed by factoring the difference and sum of cubes:

$x^6 – 64 = (x^3)^2 – 8^2$

$= (x^3 – 8)(x^3 + 8)$

$= (x^3 – 2^3)(x^3 + 2^3)$

$= (x – 2)(x^2 + 2x + 4)(x + 2)(x^2 – 2x + 4)$

$= (x – 2)(x + 2)(x^2 + 2x + 4)(x^2 – 2x + 4)$

factoring a difference of cubes, followed by factoring the difference of squares:

$x^6 – 64 = (x^2)^3 – 4^3$

$= (x^2 - 4)((x^2)^2 + 4x^2 + 4^2)$

$= (x^2 - 2^2)(x^4 + 4x^2 + 16)$

$= (x - 2)(x + 2)(x^4 + 4x^2 + 16)$

$= (x - 2)(x + 2)(x^4 + 4x^2 + 16)$

You *should* get full credit for either answer, since you shouldn't be expected to know (or somehow to guess) that the quartic polynomial:

$x^4 + 4x^2 + 16$

...factors as:

$(x^2 + 2x + 4)(x^2 - 2x + 4)$

But if you happen to notice that a problem could be worked either way (as a difference of squares or as a difference of cubes), then you can see from the above example that it might be best to apply the difference-of-squares formula first. Doing the factoring of the difference of squares first means that you'll end up getting all four factors, not just three of them.

Since the hardest part of factoring usually comes in figuring out how to proceed with a given problem, below are some factoring examples, with an explanation of which way you need to go with it to arrive at the answer.

- **Factor $x^2 + 11x + 18$**

This polynomial has three terms, and the third term, 18, isn't a square of anything, so this isn't going to be a perfect-square trinomial. So I'll first try to factor the "usual" way.

For this quadratic, I'll need to find factors of 18 that add up to 11, and then fill in the parentheses. The factors will be 9 and 2 so, filling in my parentheses, I get:

$x^2 + 11x + 18$

= **$(x + 2)(x + 9)$**

- **Factor $16x^2 – 49$**

This quadratic has two terms, and nothing factors out of both terms, so I need to be thinking "difference of squares, or sum or difference of cubes", because these are the only patterns I have for two-term quadratics.

Since there are no cubes (and especially since the variable *x* is squared), I should look for a difference of squares. Sixteen is a square, and so is 49, so I'll apply the difference of squares formula to $(4x)^2 – 7^2$:
$16x^2 – 49$
$= (4x)^2 – 7^2$

= **$(4x – 7)(4x + 7)$**

- **Factor $3x^3 – 12x$**

First, I'll see if anything factors out of both of these two terms. It turns out that I can factor out a $3x$, giving me:

$3x^3 – 12x = 3x(x^2 – 4)$

This leaves me with two terms inside the parentheses, where the two terms have a subtraction in the middle, and the *x* is squared and the second term, the "4", can be expressed as a square; namely, 2^2:

$x^2 – 4 = x^2 – 2^2$

I can then apply the difference-of-squares formula to $x^2 – 2^2$, to get:

$x^2 – 2^2 = (x – 2)(x + 2)$

I need to be careful, after factoring the difference of sqaures, that I don't forget the factor of "$3x$" that I took out first, when I write my final answer.

$3x(x – 2)(x + 2)$

- **Factor $x^2 + 6x + 9$**

This is a quadratic with three terms. I'll factor it in the "usual" way:

$x^2 + 6x + 9$

$= (x + 3)(x + 3)$

$= \mathbf{(x + 3)^2}$

You might also have noticed that this is a perfect square trinomial, from the fact that x^2 is the square of x, 9 is the square of 3, and $2(x)(3) = 6x$, which matches the middle term of the original quadratic. Notice that, had you noticed this right away, you might have shaved a few seconds off your time. Occasionally, this can prove helpful.

- **Factor $27x^3 – 8$**

This has two terms, and there's nothing common to both terms, so I can't factor anything out.

However, this binomial is a difference. It's not a difference of squares, though; it's a difference of cubes. Twenty-seven is the cube of 3, and so is 8 is the cube of 2. Therefor, I can apply the difference-of-cubes formula to $(3x)^3 – 2^3$.

$27x^3 – 8 = (3x)^3 – 2^3$

$= (3x – 2)\ ((3x)^2 + (3x)(2) + (2)^2)$

$= \mathbf{(3x – 2)(9x^2 + 6x + 4)}$

- **Factor $7x^7 – 56x$**

This has two terms, and a $7x$ comes out of both, giving me:

$7x^7 – 56x = 7x(x^6 – 8)$

Inside the parentheses, I still have two terms, and it's a difference. The first term, x^6, could be a cube, $(x^2)^3$, or a square, $(x^3)^2$, but 8 can only be a cube, 2^3. So I'll apply the difference-of-cubes formula to $(x^2)^3 – 2^3$.

$x^6 – 8 = (x^2)^3 – 2^3$

$= (x^2 – 2)\ ((x^2)^2 + (x^2)(2) + (2)^2)$

$= (x^2 – 2)(x^4 + 2x^2 + 4)$

Now I need to remember that $7x$ that I factored out at the beginning. Putting it all together, my answer is:

$7x(x^2 – 2)(x^4 + 2x^2 + 4)$

- **Factor $x^9 + 1$**

This poly has two terms, and nothing factors out of both. The power on the variable is 9, which is a multiple of 3, so this could be a cube. (It certainly cannot be a square, and sums of squares don't factor anyway, so that's off the table).

I remember that I can put any power I feel like on 1, so I just have to figure out what to do with the x^9.

Since the polynomial they gave me is a sum, not a difference, I have to hope that there is some way I can turn x^9 into a cube. There is: I can apply the sum of cubes formula to:

$x^9 + 1 = (x^3)^3 + 1^3$

...to get:

$(x^3 + 1)\ ((x^3)^2 – (x^3)(1) + (1)^2)$

$= (x^3 + 1)(x^6 – x^3 + 1)$

Taking another look before I assume that I'm finished with this exercise, I notice that the first factor is itself a sum of cubes. So I can apply the sum of cubes formula again:

$x^3 + 1 = x^3 + 1^3$

$= (x + 1)\ ((x)^2 - (x)(1) + (1)^2)$

$= (x + 1)(x^2 - x + 1)$

Putting it all together, I get a completely-factored answer of:

$\mathbf{(x + 1)(x^2 - x + 1)(x^6 - x^3 + 1)}$

- *Factor* $(x + y)^3 + (x - y)^3$

Yes, this is needlessly complex, but you might see something like this in an extra-credit assignment.

This is just a big lumpy sum of cubes. I'll need to be very careful with my parentheses when applying the sum-of-cubes formula. As you can imagine, there are many opportunities for me to make mistakes.

$(x + y)^3 + (x - y)^3$

$= [\ (x + y) + (x - y)\ ]\ [\ (x + y)^2 - (x + y)(x - y) + (x - y)^2\ ]$

$= [\ x + y + x - y\ ]\ [\ (x + y)^2 - (x^2 - y^2) + (x - y)^2\ ]$

$= [\ 2x\ ]\ [\ (x^2 + 2xy + y^2) - (x^2 - y^2) + (x^2 - 2xy + y^2)\ ]$

$= [\ 2x\ ]\ [\ x^2 + 2xy + y^2 - x^2 + y^2 + x^2 - 2xy + y^2\ ]$

$= [\ 2x\ ]\ [\ x^2 - x^2 + x^2 + 2xy - 2xy + y^2 + y^2 + y^2\ ]$

$= [\ 2x\ ]\ [\ x^2 + 3y^2\ ]$

- *Factor $x^4 + 8x^2 + 16 - y^2$.*

Um... what?

This fits absolutely no patterns I've seen. What on earth am I supposed to do with this?

Well, for a start, I can notice that the first three terms are a quadratic in just one variable; namely, x. Also, I can notice that this quadratic is a perfect-square trinomial:

$x^4 + 8x^2 + 16$

$= (x^2)^2 + 2(x^2)(4) + (4)^2$

$= (x^2 + 4)^2$

In other words, they've given me a disguised difference of squares:

$x^4 + 8x^2 + 16 - y^2$

$= (x^2 + 4)^2 - y^2$

So I can apply the difference-of-squares formula to get:

$(x^2 + 4)^2 - y^2$

$[(x^2 + 4) - y]\ [(x^2 + 4) + y]$

To successfully complete these problems, just take your time, and don't be afraid to try stuff, and to rely on your own instincts and common sense.

REFERENCES:

- PTBB General Math class 10 Chapter 1

  STBB General Math book Class 9-10 Chapter 8

UNIT 3

Surds and their Application

SLO'S:

· Recognize the surds and their application.

· Explain the surds of second order. Use basic operations on surds of second order to rationalize the denominators and evaluate it.

ASSUMED KNOWLEDGE

- Familiarity with fractions and decimals.
- Facility with basic algebra including algebraic fractions
- Familiarity with the difference of two squares and simple binomial expansions.
- Familiarity with Pythagoras' theorem.

MOTIVATION

When applying Pythagoras' theorem, irrational numbers such as $\sqrt{5}$ naturally arise. When solving a quadratic equation, using either the method of completing the square or the quadratic formula, we obtain answers such as $\frac{3+\sqrt{11}}{2}$, $\frac{3-\sqrt{11}}{2}$. These numbers involve surds. Since these numbers are irrational, we cannot express them in exact form using decimals or fractions. In some problems we may wish to approximate them using decimals, but for the most part, we prefer to leave them in exact form. Thus we need to be able to manipulate these types of numbers and simplify combinations of them which arise in the course of solving a problem. There are a number of reasons for doing this:

- approximating irrationals by decimals when problem solving can lead to rounding error. Thus it is best, if possible, to approximate at the end of a calculation and work with exact values at each step. As soon as we approximate, information is lost.
- working with exact values enables us to see important simplifications and gives further insight that would be lost if we approximate everything using decimals.
- surds give the students further practice with algebraic ideas and reinforce their basic algebra. Just as we can only combine like terms in algebra, so we can only combine like surds.
- several of the trigonometric ratios of 30°, 45° and 60° (as well as other angles) turn out to be expressible in terms of surds. Again, it is best to leave the answers in exact surd form rather than approximate. It is a lovely result in trigonometry that $\cos 72° = \frac{\sqrt{5}-1}{4}$. This is much more remarkable and gives us much better insight than simply writing $\cos 72° \approx 0.30902$, (correct to 5 decimal places)!
- the technique of *rationalising the denominator*, which is developed to handle surds in the denominator, arises in other settings in algebra, calculus and later mathematics. A similar

technique, is needed when dealing with quotients of complex numbers.

For all these reasons, an ability to manipulate and work with surds is very important for any student who intends to study mathematics at the senior level in a calculus-based or statistics course.

CONTENT

The number 9 has two square roots, 3 and −3. However, when we write $\sqrt{9}$ we always mean the positive square root, 3 and not the negative square root −3, which can be written as $-\sqrt{9}$. Every positive number has exactly two square roots. The expression $\sqrt{x}$ is only defined when x is positive or zero. For cube roots, the problem does not arise, since every number has exactly one cube root. Thus $\sqrt[3]{27} = 3$ and $\sqrt[3]{-8} = -2$. Further detail on taking roots is discussed in the module, *Indices and logarithms*.

If a is a rational number, and n is a positive integer, any irrational number of the form $\sqrt[n]{a}$ will be referred to as a surd. A real number such as $2\sqrt{3}$ will be loosely referred to as a surd, since it can be expressed as $\sqrt{12}$. For the most part, we will only consider quadratic surds, $\sqrt{a}$, that involve square roots. We will also say that $\sqrt{2} + \sqrt{3}$ is a surd, although technically we should say that it is the sum of two surds.

If a, b are positive numbers, the basic rules for square roots are:

- $(\sqrt{a})^2 = a$
- $\sqrt{a^2} = a$
- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
- $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$.

The first two of these remind us that, for positive numbers, squaring and taking a square root are **inverse processes**.

Note that these rules only work when a, b are **positive** numbers. Note that in general $\sqrt{a^2} \neq a$. Thus, the oft quoted 'conundrum' $1 = \sqrt{1} = \sqrt{(-1) \times (-1)} = \sqrt{(-1)} \times \sqrt{(-1)} = -1$ has its first error in the third equal sign. Also the $\sqrt{(-1)}$ is not defined.

Note also that the number π is not a surd. It cannot be expressed as the nth root of a rational number, or a finite combination of such numbers. Indeed, π is a transcendental number - see the module, *The Real Numbers*.

As in algebra, we write $2\sqrt{3}$ for $2 \times \sqrt{3}$.

SIMPLIFYING SURDS

In order to manipulate surds properly, we need to be able to express them in their simplest form. By simplest form, we mean that the number under the square root sign has no square factors (except of course 1). For example, the surd $\sqrt{18}$ can be simplified by writing

$\sqrt{18} = \sqrt{9 \times 2} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$.

In the second step, we used the third rule listed above. Simplifying surds enables us to identify like surds easily. (See following page for discussion of like surds.)

UNIT 4

Rationalization and Factorization

SLO'S:

- Explain rationalization (with precise meaning) of real numbers of the types , and their combinations, where x and y are natural numbers and a and b and are integers.

- Factorize the expressions of the following types:

1. Type I: (Common factors in all the terms)

   kx + ky + kz

2. Type II: ax + ay + bx + by

3. Type III: (Perfect squares)

   ± 2ab +

4. Type IV: (Difference of two squares)

5. Type V: (± 2ab +)

6. Type VI: + or +4

7. Type VII: + px + q

8. Type VIII: a + bx + c

9. Type IX: + 3 b + 3a+ and

3 b + 3a

Type X: ±

Section 1-5: Factoring Polynomials

Of all the topics covered in this chapter factoring polynomials is probably the most important topic. There are many sections in later chapters where the first step will be to factor a polynomial. So, if you can't factor the polynomial then you won't be able to even start the problem let alone finish it.

Let's start out by talking a little bit about just what factoring is. Factoring is the process by which we go about determining what we multiplied to get the given quantity. We do this all the time with numbers. For instance, here are a variety of ways to factor 12.

12=(2)(6) 12=(3)(4) 12=(2)(2)(3) 12=(12)(24) 12=(−2)(−6) 12=(−2)(2)(−3)

12=(2)(6) 12=(3)(4) 12=(2)(2)(3) 12=(12)(24) 12=(−2)(−6)
12=(−2)(2)(−3)

There are many more possible ways to factor 12, but these are representative of many of them.

A common method of factoring numbers is to **completely factor** the number into positive prime factors. A **prime** number is a number whose only positive factors are 1 and itself. For example, 2, 3, 5, and 7 are all examples of prime numbers. Examples of numbers that aren't prime are 4, 6, and 12 to pick a few.

If we completely factor a number into positive prime factors there will only be one way of doing it. That is the reason for factoring things in this way. For our example above with 12 the complete factorization is,

12= (2) (2) (3) 12= (2) (2) (3)

Factoring polynomials is done in pretty much the same manner. We determine all the terms that were multiplied together to get the given polynomial. We then try to factor each of the terms we found in the first step. This continues until we simply can't factor anymore. When we can't do any more factoring we will say that the polynomial is **completely factored.**

Here are a couple of examples.

x2−16=(x+4) (x−4) x2−16=(x+4) (x−4)

This is completely factored since neither of the two factors on the right can be further factored. Likewise,

x4−16=(x2+4) (x2−4) x4−16=(x2+4) (x2−4)

is not completely factored because the second factor can be further factored. Note that the first factor is completely factored however. Here is the complete factorization of this polynomial.

x4−16=(x2+4) (x+2) (x−2) x4−16=(x2+4) (x+2) (x−2)

The purpose of this section is to familiarize ourselves with many of the techniques for factoring polynomials.

Greatest Common Factor

The first method for factoring polynomials will be factoring out the **greatest common factor**. When factoring in general this will also be the first thing that we should try as it will often simplify the problem.

To use this method all that we do is look at all the terms and determine if there is a factor that is in common to all the terms. If there is, we will factor it out of the polynomial. Also note that in this case we are really only using the distributive law in reverse. Remember that the distributive law states that

a (b+c) =ab+ac a (b+c) =ab+ac

In factoring out the greatest common factor we do this in reverse. We notice that each term has an aa in it and so we "factor" it out using the distributive law in reverse as follows,

ab+ac=a (b+c) ab+ac=a (b+c)

Let's take a look at some examples.

Example 1 Factor out the greatest common factor from each of the following polynomials.

1. 8x4−4x3+10x28x4−4x3+10x2
2. x3y2+3x4y+5x5y3x3y2+3x4y+5x5y3
3. 3x6−9x2+3x3x6−9x2+3x
4. 9x2(2x+7)−12x(2x+7)9x2(2x+7)−12x(2x+7)

a 8x4−4x3+10x28x4−4x3+10x2
b x3y2+3x4y+5x5y3x3y2+3x4y+5x5y3
c 3x6−9x2+3x3x6−9x2+3x
d 9x2(2x+7)−12x(2x+7)9x2(2x+7)−12x(2x+7)

Factoring By Grouping

This is a method that isn't used all that often, but when it can be used it can be somewhat useful. This method is best illustrated with an example or two.

Example 2 Factor by grouping each of the following.

1. 3x2−2x+12x−83x2−2x+12x−8
2. x5+x−2x4−2x5+x−2x4−2

a 3x2−2x+12x−83x2−2x+12x−8
b x5+x−2x4−2x5+x−2x4−2
c x5−3x3−2x2+6x5−3x3−2x2+6
Factoring by grouping can be nice, but it doesn’t work all that often. Notice that as we saw in the last two parts of this example if there is a “-” in front of the third term we will often also factor that out of the third and fourth terms when we group them.

Factoring Quadratic Polynomials

First, let’s note that quadratic is another term for second degree polynomial. So we know that the largest exponent in a quadratic polynomial will be a 2. In these problems we will be attempting to factor quadratic polynomials into two first degree (hence forth linear) polynomials. Until you become good at these, we usually end up doing these by trial and error although there are a couple of processes that can make them somewhat easier.

Let’s take a look at some examples.

Example 3 Factor each of the following polynomials.

1. x2+2x−15x2+2x−15
2. x2−10x+24x2−10x+24
3. x2+6x+9x2+6x+9
4. x2+5x+1x2+5x+1
5. 3x2+2x−83x2+2x−8
6. 5x2−17x+65x2−17x+6
7. 4x2+10x−64x2+10x−6

a. x2+2x−15x2+2x−15
b. x2−10x+24x2−10x+24
c. x2+6x+9x2+6x+9
d. x2+5x+1x2+5x+1
e. 3x2+2x−83x2+2x−8
f. 5x2−17x+65x2−17x+6
g. 4x2+10x−64x2+10x−6

Special Forms

There are some nice special forms of some polynomials that can make factoring easier for us on occasion. Here are the special forms.

A^2+2ab+b2=(a+b)2

a2−2ab+b2=(a−b)2

a2−b2=(a+b)(a−b)

a3+b3=(a+b)(a2−ab+b2)

a3−b3=(a−b)(a2+ab+b2)

$a^2+2ab+b^2=(a+b)^2$

$a^2 - 2ab + b^2 = (a-b)$

$2a^2 - b^2 = (a+b)(a-b)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Let’s work some examples with these.
Example 4 Factor each of the following.

1. x2−20x+100x2−20x+100
2. 25x2−925x2−9
3. 8x3+18x3+1

a x2−20x+100x2−20x+100
b 25x2−925x2−9
c 8x3+18x3+1
Do not make the following factoring mistake!

a2+b2≠(a+b)2a2+b2≠(a+b)2

This just simply isn’t true for the vast majority of sums of squares, so be careful not to make this very common mistake. There are rare cases where this can be done, but none of those special cases will be seen here.
Factoring Polynomials with Degree Greater than 2
There is no one method for doing these in general. However, there are some that we can do so let’s take a look at a couple of examples.
Example 5 Factor each of the following.

1. 3x4−3x3−36x23x4−3x3−36x2
2. x4−25x4−25
3. x4+x2−20x4+x2−20

a. 3x4−3x3−36x23x4−3x3−36x2
b. x4−25x4−25
c. x4+x2−20x4+x2−20
We did not do a lot of problems here and we didn’t cover all the possibilities. However, we did cover some of the most common techniques that we are liable to run into in the other chapters of this work.

REFERENCES:

PTBB General Math class 10 Chapter 1

UNIT 5

Remainder Theorem and Factor Theorem

SLO'S:

- State and apply remainder theorem.
- Calculate remainder (without dividing) when a polynomial is divided by a linear polynomial.
- Define zeros of a polynomial.
- State factor theorem and explain through examples

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 2

PTBB Math class 9 Chapter 5

STBB Math 9 & 10 Chapter 5

UNIT 6

Factorization of Cubic Polynomial

SLO'S:

- Apply factor theorem to factorize a cubic polynomial.

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 2

PTBB Math class 9 Chapter 5

STBB Math 9 & 10 Chapter 5

UNIT 7

Highest Common Factor (HCF)/Greatest Common Divisor (GCD) and Least Common Multiple (LCM)

SLO'S:

- Calculate Highest Common Factor (HCF) and Least Common Multiple (LCM) of algebraic expressions by factorization method.
- Apply division method to determine highest common factor and least common multiple.
- Describe the relationship between HCF and LCM.
- Solve real life problems related to HCF and LCM.

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 3

STBB Math 9 & 10 Chapter 5

PTBB Math class 9 Chapter 6

UNIT 8

Basic Operations on Algebraic Fractions

SLO'S:

- Apply highest common factor and least common multiple to reduce fractional expressions involving addition(+), subtraction(-), multiplication(and division(

VIDEOS

REFERENCES:

NO

UNIT 9

Square Root of an Algebraic Expression

SLO'S:

- Calculate square root of an algebraic expression by factorization and division methods.

VIDEOS

REFERENCES:

NO

UNIT 10

Linear Equations

SLO'S:

- Define linear equations in one variable.
- Solve linear equations with rational coefficients.
- Convert equations, involving radicals, to simple linear form and calculate their solutions and its verification.
- solve word problems based on linear equation and verify its solutions.

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 4

PTBB Math class 9 chapter 7

UNIT 11

Equations involving Absolute Values

SLO'S:

- Define absolute value.
- Solve the equations, involving absolute values in one variable.

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 4

PTBB Math class 9 chapter 7

UNIT 12

Linear Inequalities

SLO'S:

- Define inequalities (> ,<) and (.
- Describe the properties of inequalities (i.e. trichotomy, transitive, additive and multiplicative

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 4

PTBB Math class 9 chapter 7

UNIT 13

Solving Linear Inequalities

SLO'S:

- Solve linear inequalities with real coefficient, in one variable.
- Represent the solution of linear inequalities on the number line.
- Solve linear inequalities, involving absolute value, in one variable of the following cases on the number line:

  a) 1 x 1 < 0

  b) 1 x 1 > 0

c) $1 x 1 < 1$

d) $1 x 1 > 1$

e) $1 x \; 1 < 0$, Where a is an integer.

f) $1 x \; a 1 > 0$, Where a is an integer.

- Represent the solution of the above cases on the number line.

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 4

PTBB Math class 9 chapter 7

UNIT 14

Quadratic Equations

SLO'S:

- Elucidate, then define quadratic equation in its standard form.

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 5

UNIT 15

Solution of Quadratic Equation

SLO'S:

- Solve a quadratic equation in one variable by Factorization.
- Solve a quadratic equation in one variable by Completing the squares.

REFERENCES:

PTBB General Math class 10 Chapter 5

UNIT 16

Quadratic Formula

SLO'S:

- Apply method of completing the squares to drive the quadratic formula.
- Apply quadratic formula to solve quadratic equations.
- Solve simple real life problems involving related to quadratic formula.

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 5

PTBB General Math class 10 Chapter 5

UNIT 17

Introduction to Matrices

SLO'S:

- Define of the following terms:
- A matrix with real entries and relate its rectangular layout (formation) with representation in real life as well.
- The rows and columns of a matrix.
- The order/size of a matrix.
- Equality of two matrices.

REFERENCES:

PTBB General Math class 10 Chapter 6

STBB General Math book Class 9-10 Chapter 9

UNIT 18

Types of Matrices

SLO'S:

- Define and identify of the followings:

a) Row matrix.

b) Column matrix.

c) Rectangular matrix.

d) Square matrix.

e) Zero/Null matrix.

f) Identity/Unit matrix.

g) Scalar matrix.

h) Diagonal matrix.

i) Transpose of a matrix.

j) Symmetric (upto three by three, 3 x 3).

- Skew-Symmetric matrices.

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 6

STBB General Math book Class 9-10 Chapter 9

UNIT 19

Addition and Subtraction of Matrices

SLO'S:

- Define whether the given matrices are conformable for addition and subtraction.
- Add and subtract matrices.
- Scalar multiplication of a matrix by a real number.
- Verify commutative and associative laws with respect to addition.
- Explain additive identity of a matrix.
- Calculate additive inverse of a matrix.

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 6

STBB General Math book Class 9-10 Chapter 9

UNIT 20

Multiplication of Matrices

SLO'S:

- Describes whether the given matrices are conformable for multiplication.
- Multiply two (or three) matrices.
- Verify associative law under multiplication.
- Prove that distributive laws.
- Prove that with the help of an example that commutative law with respect to multiplication does not hold, in general. (i.e., AB ≠ BA)
- Define multiplicative identity of a matrix.
- Verify the result (A =

VIDEOS

REFERENCES:

PTBB General Math class 10 Chapter 6

PTBB General Math class 10 Chapter 6

STBB General Math book Class 9-10 Chapter 9

UNIT 21

Determinant of a Matrix

SLO'S:

- Define the determinant of square matrix.

- Evaluate determinant of matrix.

- Describe the followings:

  Singular and Nonsingular matrices.

- Define adjoint of matrix.

·Calculate the multiplicative inverse of a non-singular matrix ‘ A’ and verify that A−1 = I = −1A , where, I is the identity matrix.

- Apply the adjoint method to calculate inverse of a non- singular matrix.

·Prove that (A)−1 = −1−1 . by the help an example.

VIDEOS

REFERENCES:

STBB General Math book Class 9-10 Chapter 9

UNIT 22

Solution of Simultaneous Linear Equations

SLO’S:

- Solve the system of two linear equations, related to real life problems, in two unknowns using:

a)Matrix Inversion Method.

b) Cramer’s Rule.

VIDEOS

REFERENCES:

NO

UNIT 23

Properties of Angles

SLO'S:

- Define adjacent, complementary, and supplementary angles.
- Define vertically –opposite angles.
- Calculate the followings:
  - a) Adjacent angles.
  - b) Complementary angle.
  - c) Supplementary angle.
  - d) Vertically Opposite angles.
  - Calculate unknown angle of a triangle.

VIDEOS

REFERENCES:

STBB general Math 9 & 10 Chapter 10

STBB Math 9 & 10 Chapter 12

PTBB general math class 10 chapter 7

UNIT 24

Parallel Lines

SLO'S:

- Define parallel line.

- Demonstrate through figures the following
- properties of parallel lines.

    a) Two lines which are parallel to the same given line are parallel to each other.

    b) If three parallel lines are intersected by two transversals in such a way that two intercepts on one transversal are equal to each other , the two intercepts on the second transversal are also equal.

    c) A line through the midpoint of a side of a triangle parallel to another side bisects the third side (an application of above property).

- Draw a transversal to intersect two parallel lines and demonstrate corresponding angles, alternate-interior angles, vertically –opposite angles and interior angles on the same side of transversal.
- Describe the following relation between the pairs of angles when a transversal intersects two parallel lines:

    a) Pairs of corresponding angles are equal.

    b) Pairs of alternate interior angles are equal.

    - Pairs of interior angles on the same side of transversal is supplementary, and demonstrate them through figures.

VIDEOS

REFERENCES:

STBB general Math 9 & 10 Chapter 10

STBB Math 9 & 10 Chapter 12

PTBB general math class 10 chapter 7

UNIT 25

Congruent and Similar Figures

SLO'S:

- Identify congruent and similar figures.
- Recognize the symbol of congruency.
- Apply the properties for two figures to congruent or similar.

VIDEOS

REFERENCES:

STBB general Math 9 & 10 Chapter 10

STBB Math 9 & 10 Chapter 12

PTBB general math class 10 chapter 7

UNIT 26

Congruent Triangles

SLO'S:

- Apply following properties for congruency between two triangles:
  a) SSS $\cong SSS$
  b) SAS $\cong SAS$
  c) ASA $\cong ASA$
  d) RHS

VIDEOS

REFERENCES:

STBB general Math 9 & 10 Chapter 10

STBB Math 9 & 10 Chapter 12

PTBB general math class 10 chapter 7

UNIT 27

Quadrilaterals

SLO'S:

- Demonstrate the following properties of a square:

a) The four sides of a square are equal.

b) The four angles of a square are right angles.

c) Diagonals of a square bisect each other and are equal.

- **Demonstrate the following properties of a parallelogram:**

a) Opposite side of a parallelogram are equal.

b) Opposite angles of a parallelogram are equal.

c) Diagonals of a parallelogram bisect each other.

VIDEOS

REFERENCES:

STBB Math 9 & 10 Chapter 12

PTBB general math class 10 chapter 7

UNIT 28

Circle

SLO'S:

- Describe the following:
  a) A circle and its centre.
  b) Radius.
  c) Diameter.
  d) Chord.
  e) Arc.
  f) Major arcs.

g) Minor arcs.

h) Semicircle

i) Segment of the circle.

- Describe the terms:

a) Sector and secant of a circle.

b) Concyclic points.

c) Tangent to a circle.

d) Concentric circles.

- Demonstrate the following properties:

a) The angle in a semicircle is a right angle.

b) The angles in the same segment of a circle are equal.

c) The central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.

- Apply the above properties in different geometrical figures.

VIDEOS

REFERENCES:

STBB Math 9 & 10 Chapter 13

STBB general Math 9 & 10 Chapter 11

PTBB general math class 10 chapter 7

PTBB Math class 10 chapter 9

STBB Math 9 & 10 Chapter 13

STBB general Math 9 & 10 Chapter 11

UNIT 29

Construction of Quadrilateral

SLO'S:

- Construct a rectangle when:
  a) Two sides are given.
  b) Diagonal and one side are given.
- Construct a square when its diagonal is given.
- Construct a parallelogram when two adjacent sides and the angle included between them is given.

VIDEOS

REFERENCES:

PTBB general math class 10 chapter 8

UNIT 30

Tangents to the Circle

SLO'S:

- Locate the centre of a given circle.
- Draw a circle passing through three given non-collinear points.
- Draw a tangent to a given circle from a point P when P lies:
  a) On the circumference.
  b) Outside the circle.
- Draw the followings:

a) Direct common tangent or external tangent

b) Transverse common tangent or internal tangent to two equal circles.

- Draw the followings:

a) Direct common tangent or external tangent.

b) Transverse common tangent or internal tangent to two Unequal circles.

- Draw a tangent to:

a) Two unequal touching circles.

b) Two unequal intersecting circles.

VIDEOS

REFERENCES:

STBB Math 9 & 10 Chapter 14

STBB general Math 9 & 10 Chapter 11

PTBB math class 10 chapter 13

UNIT 31

Pythagoras Theorem

SLO'S:

i) State Pythagoras theorem.

ii) Solve right angle triangle by using Pythagoras theorem.

What is the Pythagorean Theorem?

You can learn all about the Pythagorean Theorem, but here is a quick summary:

The Pythagorean Theorem says that, *in a right triangle*, the square of a (a^2) plus the square of b

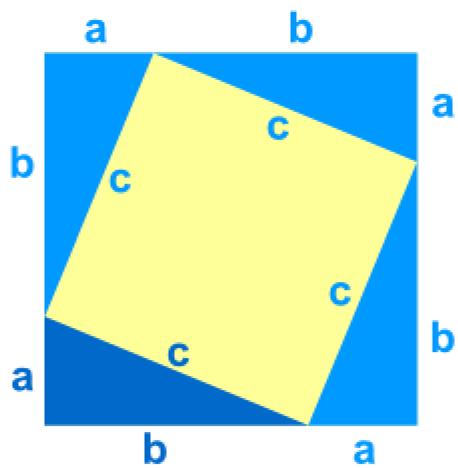
(b^2) is equal to the square of c (c^2):

$$a^2 + b^2 = c^2$$

Proof of the Pythagorean Theorem using Algebra

We can show that $\mathbf{a^2 + b^2 = c^2}$ using Algebra

Take a look at this diagram ... it has that "abc" triangle in it (four of them actually):



Area of Whole Square

It is a big square, with each side having a length of **a+b**, so the **total area** is:

$$A = (a+b)(a+b)$$

Area of the Pieces

Now let's add up the areas of all the smaller pieces:

First, the smaller (tilted) square has an area of: c^2

Each of the four triangles has an area of: $\frac{ab}{2}$

So all four of them together is: $4\frac{ab}{2} = 2ab$

Adding up the tilted square and the 4 triangles gives: $A = c^2 + 2ab$

Both Areas Must Be Equal

The area of the **large square** is equal to the area of the **tilted square and the 4 triangles**. This can be written as:

$$(a+b)(a+b) = c^2 + 2ab$$

NOW, let us rearrange this to see if we can get the pythagoras theorem:

Start with: $(a+b)(a+b) = c^2 + 2ab$

Expand (a+b)(a+b): $a^2 + 2ab + b^2 = c^2 + 2ab$

Subtract "2ab" from both sides: $a^2 + b^2 = c^2$

DONE!

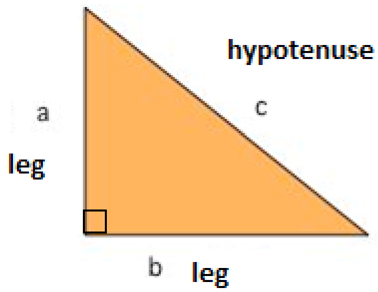
Now we can see why the Pythagorean Theorem works ... and it is actually a **proof** of the Pythagorean Theorem.

This proof came from China over 2000 years ago!

There are many more proofs of the Pythagorean theorem, but this one works nicely.

The Pythagorean Theorem

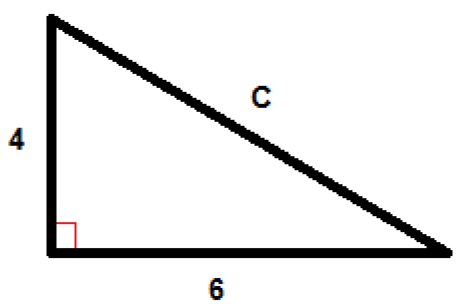
One of the best known mathematical formulas is Pythagorean Theorem, which provides us with the relationship between the sides in a right triangle. A right triangle consists of two legs and a hypotenuse. The two legs meet at a 90° angle and the hypotenuse is the longest side of the right triangle and is the side opposite the right angle.



The Pythagorean Theorem tells us that the relationship in every right triangle is:

a2+b2=c2a2+b2=c2

Example



C2=62+42C2=62+42

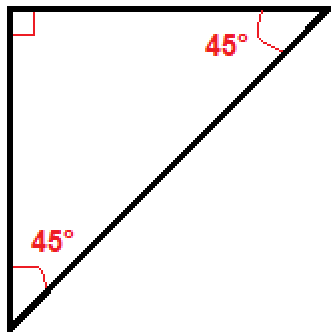
C2=36+16C2=36+16

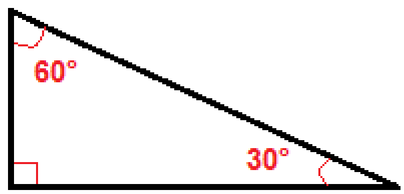
C2=52C2=52

C=52−−√C=52

C≈7.2C≈7.2

There are a couple of special types of right triangles, like the 45°-45° right triangles and the 30°-60° right triangle.

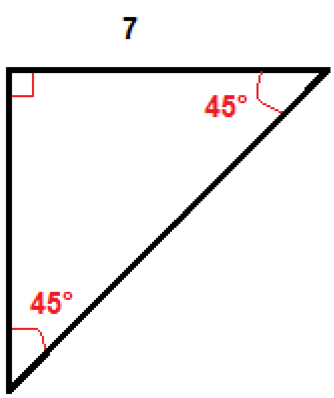




Because of their angles it is easier to find the hypotenuse or the legs in these right triangles than in all other right triangles.

In a 45°-45° right triangle we only need to multiply one leg by √2 to get the length of the hypotenuse.

Example



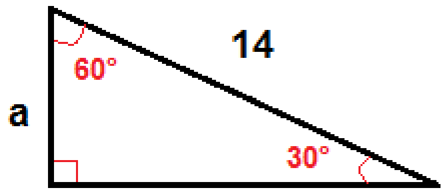
We multiply the length of the leg which is 7 inches by √2 to get the length of the hypotenuse.

7·2–√≈9.97·2≈9.9

In a 30°-60° right triangle we can find the length of the leg that is opposite the 30° angle by using this formula:

a=12·ca=12·c

Example



To find a, we use the formula above.

$$a=12\cdot 14a=12\cdot 14$$

$$a=7$$

The Pythagorean Theorem

Learning Objective(s)

- Use the Pythagorean Theorem to find the unknown side of a right triangle.
- Solve application problems involving the Pythagorean Theorem.

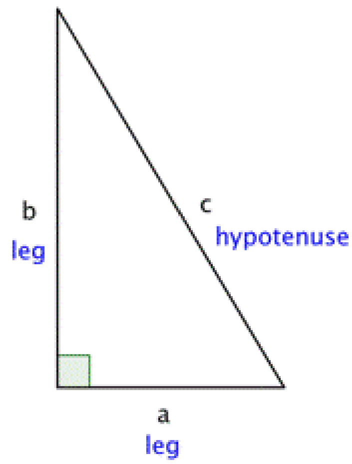
Introduction

A long time ago, a Greek mathematician named **Pythagoras** discovered an interesting property about **right triangles**: the sum of the squares of the lengths of each of the triangle's **legs** is the same as the square of the length of the triangle's **hypotenuse**. This property—which has many applications in science, art, engineering, and architecture—is now called the **Pythagorean Theorem**.

Let's take a look at how this theorem can help you learn more about the construction of triangles. And the best part—you don't even have to speak Greek to apply Pythagoras' discovery.

The Pythagorean Theorem

Pythagoras studied right triangles, and the relationships between the legs and the hypotenuse of a right triangle, before deriving his theory.



The Pythagorean Theorem

If *a* and *b* are the lengths of the legs of a right triangle and *c* is the length of the hypotenuse, then the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

This relationship is represented by the formula: $a^2 + b^2 = c^2$

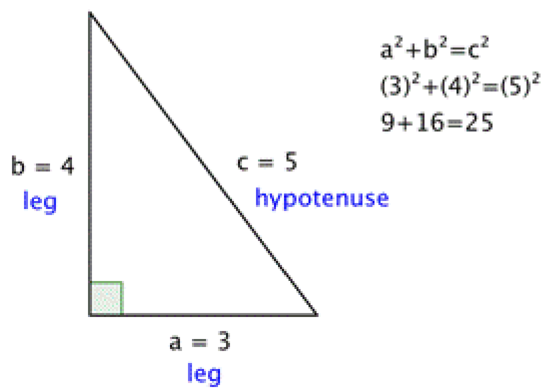
In the box above, you may have noticed the word "square," as well as the small 2s to the top right of the letters in $a^2 + b^2 = c^2$. To **square** a number means to multiply it by itself. So, for example, to square the number 5 you multiply 5 • 5, and to square the number 12, you multiply 12 • 12. Some common squares are shown in the table below.

Number	Number Times Itself	Square
1	$1^2 = 1 \cdot 1$	1
2	$2^2 = 2 \cdot 2$	4
3	$3^2 = 3 \cdot 3$	9

4	$4^2 = 4 \cdot 4$	16
5	$5^2 = 5 \cdot 5$	25
10	$10^2 = 10 \cdot 10$	100

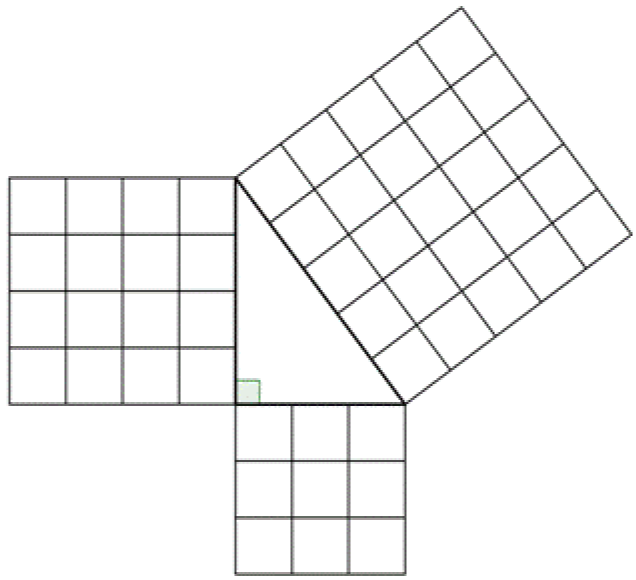
When you see the equation $a^2 + b^2 = c^2$, you can think of this as "the length of side *a* times itself, plus the length of side *b* times itself is the same as the length of side *c* times itself."

Let's try out all of the Pythagorean Theorem with an actual right triangle.



This theorem holds true for this right triangle—the sum of the squares of the lengths of both legs is the same as the square of the length of the hypotenuse. And, in fact, it holds true for all right triangles.

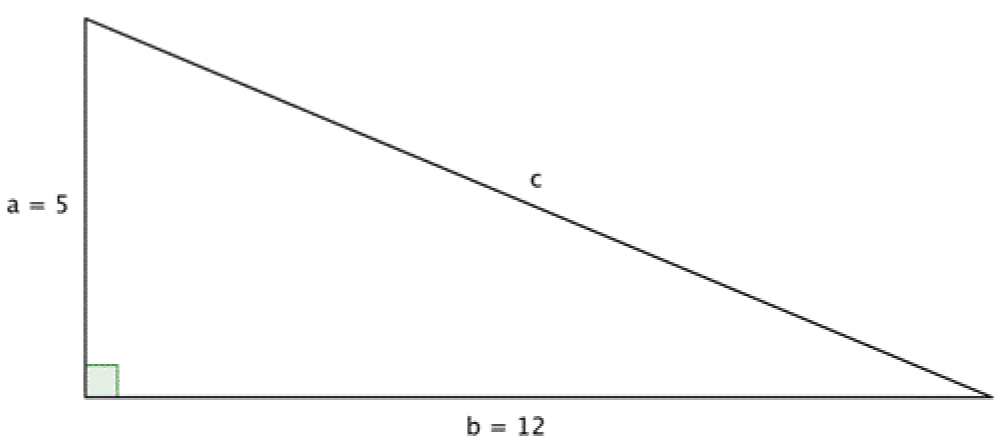
The Pythagorean Theorem can also be represented in terms of area. In any right triangle, the area of the square drawn from the hypotenuse is equal to the sum of the areas of the squares that are drawn from the two legs. You can see this illustrated below in the same 3-4-5 right triangle.



Note that the Pythagorean Theorem only works with *right* triangles.

Finding the Length of the Hypotenuse

You can use the Pythagorean Theorem to find the length of the hypotenuse of a right triangle if you know the length of the triangle's other two sides, called the legs. Put another way, if you know the lengths of *a* and *b*, you can find *c*.



In the triangle above, you are given measures for legs *a* and *b*: 5 and 12, respectively. You can use the Pythagorean Theorem to find a value for the length of *c*, the hypotenuse.

$a^2 + b^2 = c^2$	The Pythagorean Theorem.
$(5)^2 + (12)^2 = c^2$	Substitute known values for *a* and *b*.
$25 + 144 = c^2$	Evaluate.
$169 = c^2$	Simplify. To find the value of *c*, think about a number that, when multiplied by itself, equals 169. Does 10 work? How about 11? 12? 13? (You can use a calculator to multiply if the numbers are unfamiliar.)
$13 = c$	The square root of 169 is 13.

Using the formula, you find that the length of *c*, the hypotenuse, is 13.

In this case, you did not know the value of *c*—you were given the square of the length of the hypotenuse, and had to figure it out from there. When you are given an equation like $169 = c^2$ and are asked to find the value of *c*, this is called finding the **square root** of a number. (Notice you found a number, *c*, whose square was 169.)

Finding a square root takes some practice, but it also takes knowledge of multiplication, division, and a little bit of trial and error. Look at the table below.

Number *x*	**Number *y* which, when multiplied by itself, equals number *x***	**Square root *y***
1	1 • 1	1
4	2 • 2	2
9	3 • 3	3

16	4 • 4	4
25	5 • 5	5
100	10 • 10	10

It is a good habit to become familiar with the squares of the numbers from 0–10, as these arise frequently in mathematics. If you can remember those square numbers—or if you can use a calculator to find them—then finding many common square roots will be just a matter of recall.

$a^2 + b^2 = 10^2 + 24^2 = 100 + 576 = \mathbf{676}$
$c^2 = 26^2 = \mathbf{676}$

They are equal, so ...

Yes, it does have a Right Angle!

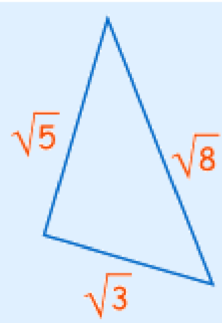
Example: Does an 8, 15, 16 triangle have a Right Angle?

Does $8^2 + 15^2 = 16^2$?

$8^2 + 15^2 = 64 + 225 = \mathbf{289}$,
but $16^2 = \mathbf{256}$

So, NO, it does not have a Right Angle

Example: Does this triangle have a Right Angle?



Does $a^2 + b^2 = c^2$?

Does $(\sqrt{3})^2 + (\sqrt{5})^2 = (\sqrt{8})^2$?

Does 3 + 5 = 8 ?

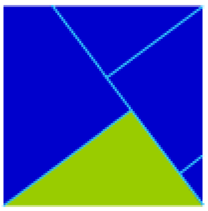
Yes, it does!

So this **is** a right-angled triangle

And You Can Prove The Theorem Yourself !

Get paper pen and scissors, then using the following animation as a guide:

Draw a right angled triangle on the paper, leaving plenty of space.
Draw a square along the hypotenuse (the longest side)
Draw the same sized square on the other side of the hypotenuse
Draw lines as shown on the animation, like this:



Cut out the shapes
Arrange them so that you can prove that the big square has the same area as the two squares on the other sides

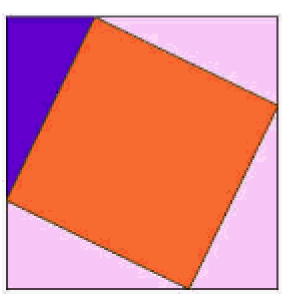
Another, Amazingly Simple, Proof

Here is one of the oldest proofs that the square on the long side has the same area as the other squares.

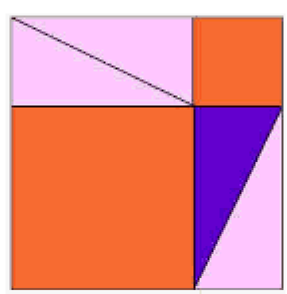
Watch the animation, and pay attention when the triangles start sliding around.

You may want to watch the animation a few times to understand what is happening.

The purple triangle is the important one.



becomes



We also have a proof by adding up the areas .

Historical Note: while we call it Pythagoras' Theorem, it was also known by Indian, Greek, Chinese and Babylonian mathematicians well before he lived !

VIDEOS

REFERENCES:

STBB general math class 9 & 10 chapter 11

UNIT 32

TOPIC: Area

SLO'S:

- Calculate the area of the following:

a) A triangle when three sides are given (apply Hero's formula).

b) A triangle whose base and altitude (height) are given.

c) An equilateral triangle when its on side is given.

d) A rectangle when its two adjacent sides are given.

e) A parallelogram when base and altitude(height) are given.

f) A square when its one side is given.

g) Four walls of a room when its length, width and height are given.

- Find the cost of turfing a square/rectangular field.
- iii) Find the number of tiles, of given dimensions, required to pave the footpath of given

width carried around the outside of a rectangular plot.

- Find the area of a circle and a semi circle when radius is given.
- Find the area enclosed by two concentric circles whose radii are given.
- Solve real life problems related with areas of triangle, rectangle, square, parallelogram and circle.

Heron's Formula for the area of a triangle(Hero's Formula)

A method for calculating the area of a triangle when you know the lengths of all three sides.

Let a,b,c be the lengths of the sides of a triangle. The area is given by:

$$Area=\sqrt{(p-a)(p-b)(p-c)}$$

where p is half the perimeter, or $\frac{a+b+c}{2}$

Try this Drag the orange dots to reshape the triangle. The formula shown will re-calculate the triangle's area using Heron's Formula

$$p=\frac{24.0+30.0+18.0}{2}=36.0$$

$$Area=\sqrt{36.0\times12.0\times6.0\times18.0}=216.00$$

Heron was one of the great mathematicians of antiquity and came up with this formula sometime in the first century BC, although it may have been known earlier. He also extended it to the area of quadrilaterals and higher-order polygons.

Calculator

Use the calculator on below to calculate the area of a triangle given 3 sides using Heron's formula.

Enter the three side lengths and press 'Calculate'. The area will be calculated.

What is Area?

Area is the size of a surface!

Example:

These shapes all have the same area of 9:

It helps to imagine **how much paint** would cover the shape.

Area of Simple Shapes

There are special formulas for certain shapes:

Example: What is the area of this rectangle?

The formula is:

Area = w × h
w = width
h = height

The width is 5, and the height is 3, so we know **w = 5** and **h = 3**:

Area = 5 × 3 = **15**

Learn more at Area of Plane Shapes.

Area by Counting Squares

We can also put the shape on a grid and count the number of squares:

The rectangle has an area of **15**

Example: When each square is **1 cm** on a side, then the area is **15** $\mathbf{cm^2}$ (15 square cm)

Approximate Area by Counting Squares

Sometimes the squares don't match the shape exactly, but we can get an "approximate" answer.

One way is:

more than half a square counts as **1**

less than half a square counts as **0**

Like this:

This pentagon has an area of **approximately 17**

Or we can count one square when the **areas seem to add up**.

Example: Here the area marked "**4**" seems equal to about 1 whole square (also for "**8**"):

This circle has an area of **approximately 14**

But using a formula (when possible) is best:

Example: The circle has a radius of 2.1 meters:

The formula is:

$$\text{Area} = \pi \times r^2$$

Where:

π = the number pi (3.1416...)

r = radius

The radius is **2.1m**, so:

$$\text{Area} = 3.1416... \times (2.1\text{m})^2$$
$$= 3.1416... \times (2.1\text{m} \times 2.1\text{m})$$
$$= 13.854...\ \text{m}^2$$

So the circle has an area of **13.85 square meters** (to 2 decimal places)

Area of Difficult Shapes

We can sometimes break a shape up into two or more simpler shapes:

Example: What is the area of this Shape?

Let's break the area into two parts:
Part A is a square:
Area of A = a^2 = 20m × 20m = 400m^2
Part B is a triangle. Viewed sideways it has a base of 20m and a height of 14m.
Area of B = ½b × h = ½ × 20m × 14m = 140m^2
So the total area is:

Area = Area of A + Area of B
Area = 400m^2 + 140m^2
Area = 540m^2

Area by Adding Up Triangles

We can also break up a shape into triangles:

Then measure the base (**b**) and height (**h**) of each triangle:

Then calculate each area (using Area = ½b × h) and add them all up.

Area by Coordinates

When we know the coordinates of each corner point we can use the Area of Irregular Polygons method.

There is an Area of a Polygon by Drawing Tool that can help too.

Heron's Formula

Area of a Triangle from Sides

You can calculate the area of a triangle if you know the lengths of all three sides, using a formula that has been known for nearly 2000 years.

It is called "Heron's Formula" after Hero of Alexandria (see below)

Just use this two step process:

Step 1: Calculate **"s"** (half of the triangles perimeter):

s = *a*+*b*+*c***2**

Step 2: Then calculate the **Area**:

Example: What is the area of a triangle where every side is 5 long?

Step 1: s = *5*+*5*+*5***2** = 7.5
Step 2: A = √(7.5 × 2.5 × 2.5 × 2.5) = √(117.1875) = **10.825...**

Try it yourself:

Heron's Formula

Find a Triangle's Area from its Sides

Classic Heron's Formula:

s = (a+b+c)/2 = 6

Area = √(s(s-a)(s-b)(s-c))

Area = 6

Variation with less rounding error:

Sides in Descending Order: 5,4,3

Area = √((a+(b+c))(c-(a-b))(c+(a-b))(a+(b-c)))/4

Area = 6

a:53.1301°

b:36.8699°

c:90°

Hero of Alexandria

The formula is credited to Hero (or Heron) of Alexandria, who was a Greek Engineer and Mathematician in 10 – 70 AD.

Amongst other things, he developed the ***Aeolipile***, the first known steam engine, but it was treated as a toy!

Angles

In the calculator above I also used the Law of Cosines to calculate the angles (for a complete solution). The formula is:

Where "C" is the angle **opposite** side "c".

The **Formula**

Heron's formula is named after Hero of Alexandria, a Greek Engineer and Mathematician in 10 - 70 AD. You can use this formula to find the area of a triangle using the 3 side lengths.

Therefore, you do not have to rely on the formula for area that uses base and height. Diagram 1 below illustrates the general formula where S represents the semi-perimeter of the triangle.

semi-perimeter is just the perimeter divided by 2 :perimeter2perimeter2 .

Diagram 1

Diagram 2

A specific example

Examples

Example 1

(Straight forward example)

Use Heron's formula to find the area of triangle ABC, if AB=3,BC=2,CA=4AB=3,BC=2,CA=4 .

Step 1

Calculate the semi perimeter, *S*

s=3+2+42s=4.5s=3+2+42s=4.5

Step 2

Substitute S into the formula .

Round answer to nearest tenth.

A=4.5(4.5−3)(4.5−2)(4.5−4)

√A=8.4375

√A≈2.9A=4.5(4.5−3)(4.5−2)(4.5−4)A=8.4375A≈2.9

Since Heron's formula relates the side lengths, perimeter and area of a triangle, you might need to answer more challenging question types like the following example.

Example 2

Given △ABC△ABC, with an area of 8.948.94 square units, a perimeter of 1616 units and side lengths AB=3AB=3 and CA=7CA=7, what is BCBC ?

Step 1

Calculate the semi perimeter, *S*.

S=perimeter2S=162S=8S=perimeter2S=162S=8

Step 2

Substitute known values into the formula . Let x=BCx=BC

A=S(S−AB)(S−BC)(S−CA)

√8.94=8(8−3)(8−x)(8−7)

√8.94=8(5)(8−x)(1)

√A=S(S−AB)(S−BC)(S−CA)8.94=8(8−3)(8−x)(8−7)8.94=8(5)(8−x)(1)

Step 3

Solve for xx (square both sides and go from there).

8.94^2=√{8(5)(8−x)(1)}^{2}

79.9236=8(5)(8−x)(1)

79.9236=40(8−x)

79.923640=8−x

1.999809=8−x

x≈6.0

8.942=(8(5)(8−x)(1)))279.9236=8(5)(8−x)(1)79.9236=40(8−x)79.923640=8−x1.999809=8−xx≈6.0

UNIT 33

Volumes

SLO'S:

- Find the volume of the followings:

  a) a cube when its edge is given.

  b) a cuboid when its length, breadth and height are given.

  c) a right circular cylinder whose base radius and height are given.

  d) a right circular cone whose radius and height are known.

  e) a sphere and a hemisphere when radius is given.

  - Solve real life problems related to volume of cube, cuboid, cylinder, cone and sphere.

3D Shapes and Volume

A three-dimensional (3D) shape has three dimensions: length, width, and height. A 2D shape has only length and width.

There are several 3D shapes, several of which can be put in one of these categories: **prism** or **pyramid**:

- **Prism**: a prism has two congruent shapes at opposite ends, parallel to each other, and connected by rectangular faces
- **Pyramid**: a pyramid has a basic shape on the base, with triangular faces that come to a point

Depending on the shape at the ends/base of these two basic 3D shapes, we can generate a whole bunch of prisms and pyramids.

Note how the name is determined by the shape of the base: if the base is a square and the sides are triangles that come to a point, it is a square pyramid. If the two shapes at the ends are squares, parallel to each other and connected by rectangles, it is a square prism.

There are two special 3D shapes whose names don't include the word "prism" even though they technically are prisms:

- **Cube**: a cube is essentially a prism (two squares opposite and parallel to each other, connected by rectangles), except that the connecting rectangles are actually squares. Cubes have six faces, all squares.
- **Cylinder**: a cylinder is a prism because it has two circles opposite and parallel to each other, which are connected by a rectangle. It's not as obvious as other prisms, but think of a can of peas: if you take the label off, it is the shape of a rectangle.

3D shapes have volume: the amount of cubic space inside of them.

To find volume, you basically need the three dimensions: length, width, and height.

For **prisms**, the formulas are derived by taking the area of the shape at the end, and multiplying that times the figure's height.

- **Rectangular Prism**: length x width x height
- **Cube**: length x width x height OR side x side x side (since they are all the same)
- **Triangular Prism**: (base x height ÷ 2) x height*
  - **this height is the height of the prism*
- **Cylinder**: (π x r x r) x height

For **pyramids**, the formulas are almost the same as for prisms, only they are divided by 3.

- **Square or Rectangular Pyramid**: (length x width x height) ÷ 3
- **Triangular Pyramid**: ((base x height ÷ 2) x height*) ÷ 3
  - **this height is the height of the prism*
- **Cone**: (π x r x r x height) ÷ 3

Examples math problems related to volume:

Example math problem related to Prisms:

1. Find the volume of this **rectangular prism**.
   - Recall the formula: length x width x height
   - (It won't change things if you're not sure which part is which, but usually length is the long side, width is the short side, and height is how tall it is.)
   - Plug in the numbers to the formula:
   - L x W x H = 12 x 4 x 6 = 288
   - To label volume, we use cubic units.
   - This rectangular prism has a volume of 288 cm3.
2. Find the volume of this **cube**.
   - Recall the formula: length x width x height OR side x side x side
   - Notice that only one side is labeled. This is okay since all the sides are the same. We know that length, width, and height are all 5 cm.
   - Plug the numbers into the formula:
   - L x W x H (or s x s x s) = 5 x 5 x 5 = 125
   - This cube has a volume of 125 cm3.
3. Find the volume of this **triangular prism**.
   - Recall the formula: (base x height ÷ 2) x height
   - First, let's identify each part.
   - The base is the "bottom" of one of the triangles: 7
   - The height is the height of one of the triangles: 5*
   - (If you get the 7 and 5 backwards, that's okay; just make sure you use numbers that are perpendicular to each other. Note that the other side of the triangle is not labeled. This side is not needed to find volume.)
   - The second height is the height of the prism: 6
   - Now plug in the numbers:
   - (base x height ÷ 2) x height = (7 x 5 ÷ 2) x 6 = 105
   - The volume of this triangular prism is 105 ft3.
4. Find the volume of this **cylinder**.
   - Recall the formula: (π x r x r) x height
   - We only need to know two things: radius (from the center of the circle to the edge) and the height of the cylinder. Remember that π = 3.14.
   - Plug in the numbers:
   - (π x r x r) x height = (3.14 x 3 x 3) x 10 = 282.6
   - The volume of this cylinder is 282.6 ft3.

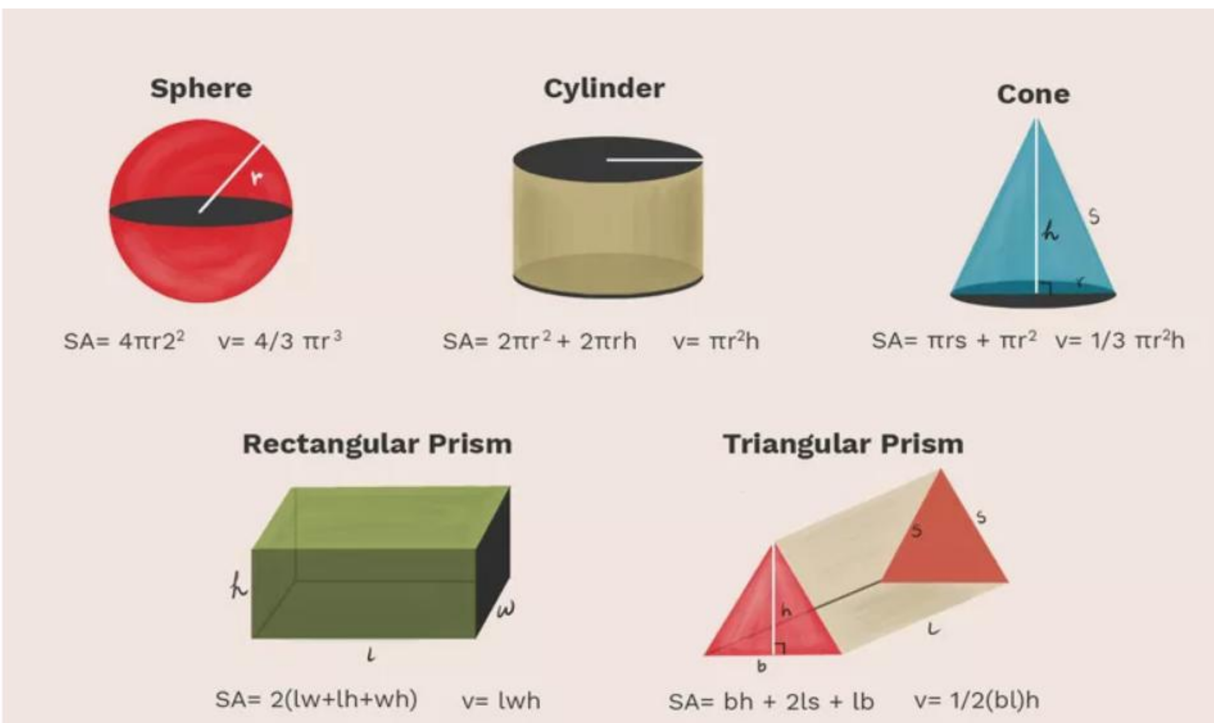
Example math problems related to pyramids:

1. Find the volume of this **square pyramid**.
   - Recall the formula: (length x width x height) ÷ 3
   - Remember, for a square, length, width, and height are all the same.
   - Plug in the numbers:
   - (length x width x height) ÷ 3 = (2 x 2 x 9) ÷ 3 = 12
   - The volume of this square pyramid is 12 cm3.
2. Find the volume of this **triangular pyramid**.

- Recall the formula: ((base x height ÷ 2) x height*) ÷ 3
- Remember that the first height is the height of the triangle base, and the second height is the height of the pyramid, perpendicular to the base.
- Plug in the numbers:
- ((base x height ÷ 2) x height*) ÷ 3 = ((7 x 8 ÷ 2) x 9) = 252
- The volume of this triangular pyramid is 252 cm3.

3. Find the volume of this **cone**.
    - Recall the formula: (π x r x r x height) ÷ 3
    - A cone has a circular base with a pointy top. All we need is the radius and the height of the cone to find its volume.
    - Plug in the numbers:
    - (π x r x r x height) ÷ 3 = (3.14 x 4 x 4 x 17) ÷ 3 = 284.69
    - The volume of this cone is 284.69 in3.

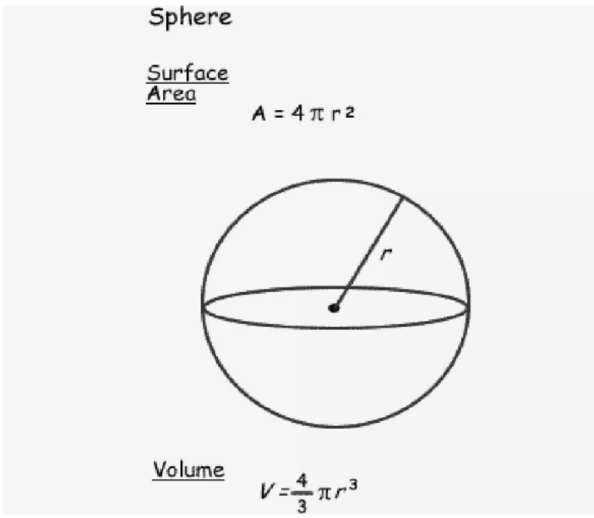
Math Formulas for Geometric Shapes



In math (especially geometry) and science, you will often need to calculate the surface area, volume, or perimeter of a variety of shapes. Whether it's a sphere or a circle, a rectangle or a cube, a pyramid or a triangle, each shape has specific formulas that you must follow to get the correct measurements.

We're going to examine the formulas you will need to figure out the surface area and volume of three-dimensional shapes as well as the area and perimeter of two-dimensional shapes. You can study this lesson to learn each formula, then keep it around for a quick reference next time you need it. The good news is that each formula uses many of the same basic measurements, so learning each new one gets a little easier.

Surface Area and Volume of a Sphere

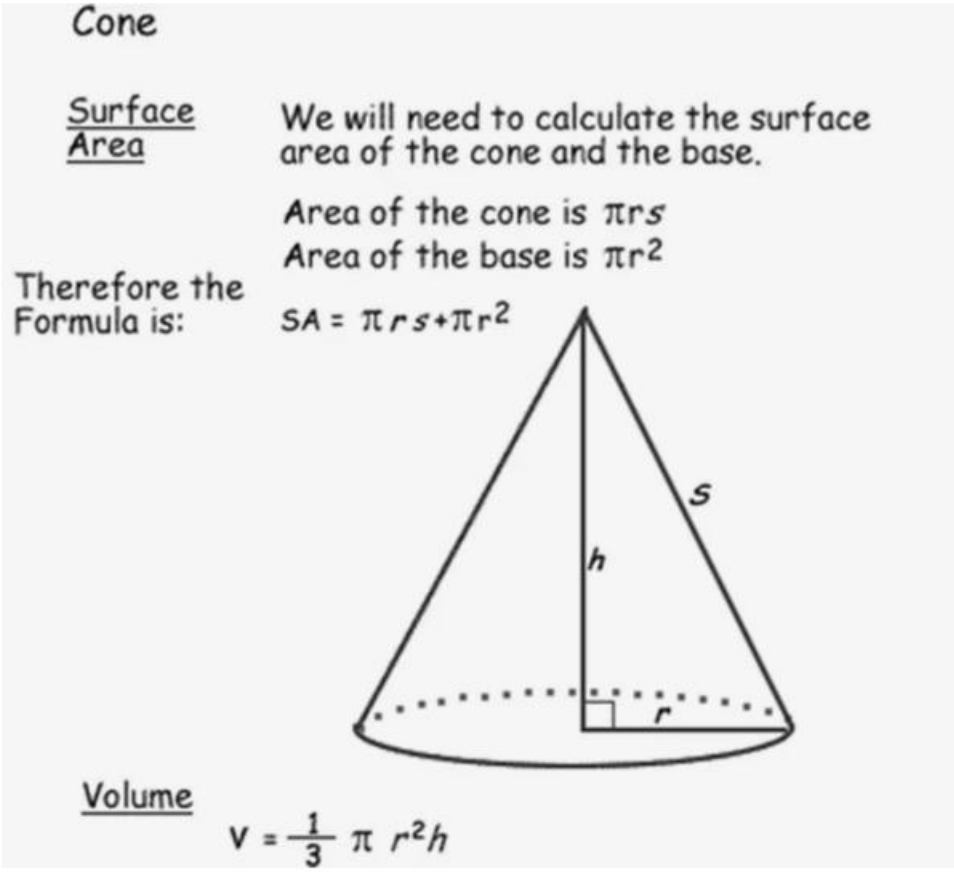


A three-dimensional circle is known as a sphere. In order to calculate either the surface area or the volume of a sphere, you need to know the radius (**r**). The radius is the distance from the center of the sphere to the edge and it is always the same, no matter which points on the sphere's edge you measure from.

Once you have the radius, the formulas are rather simple to remember. Just as with the circumference of the circle, you will need to use pi (**π**). Generally, you can round this infinite number to 3.14 or 3.14159 (the accepted fraction is 22/7).

- **Surface Area = $4\pi r^2$**
- **Volume = $4/3\ \pi r^3$**

02. Surface Area and Volume of a Cone



A cone is a pyramid with a circular base that has sloping sides which meet at a central point. In order to calculate its surface area or volume, you must know the radius of the base and the length of the side.

If you do not know it, you can find the side length (**s**) using the radius (**r**) and the cone's height (**h**).

- **s = √(r2 + h2)**

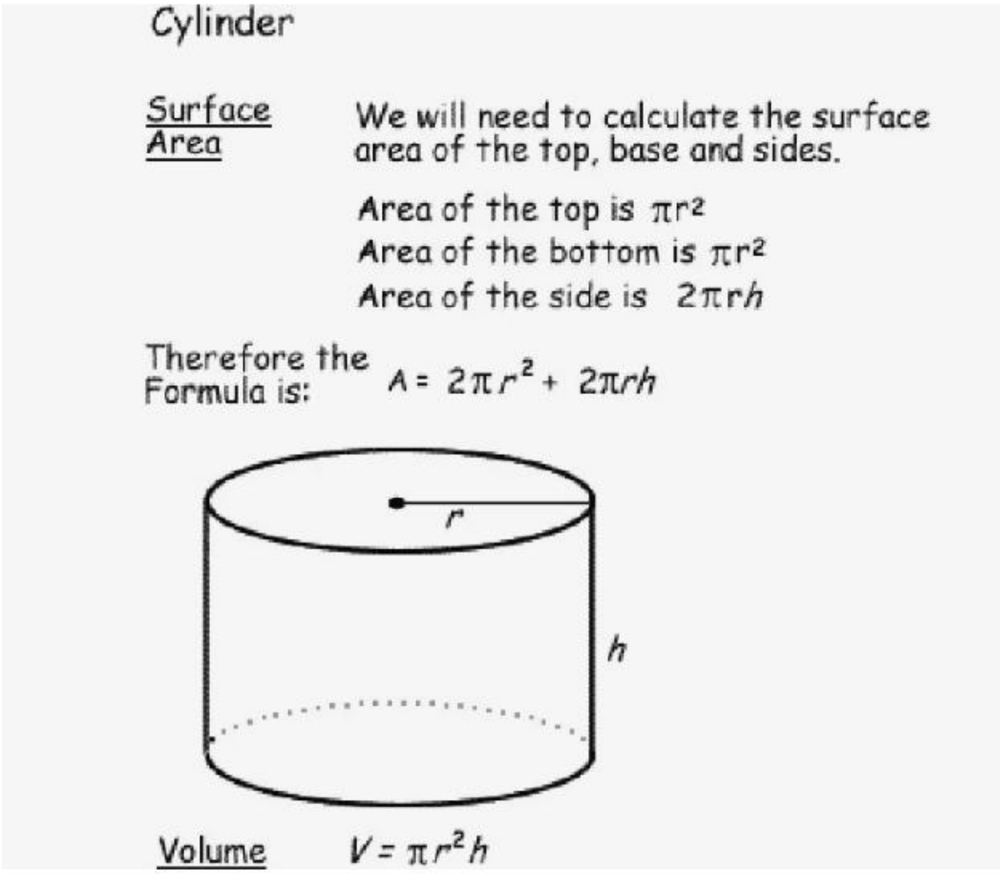
With that, you can then find the total surface area, which is the sum of the area of the base and area of the side.

- **Area of Base: πr^2**
- **Area of Side: πrs**
- **Total Surface Area = $\pi r^2 + \pi rs$**

To find the volume of a sphere, you only need the radius and the height.

- **Volume = $1/3\ \pi r^2 h$**

03. Surface Area and Volume of a Cylinder

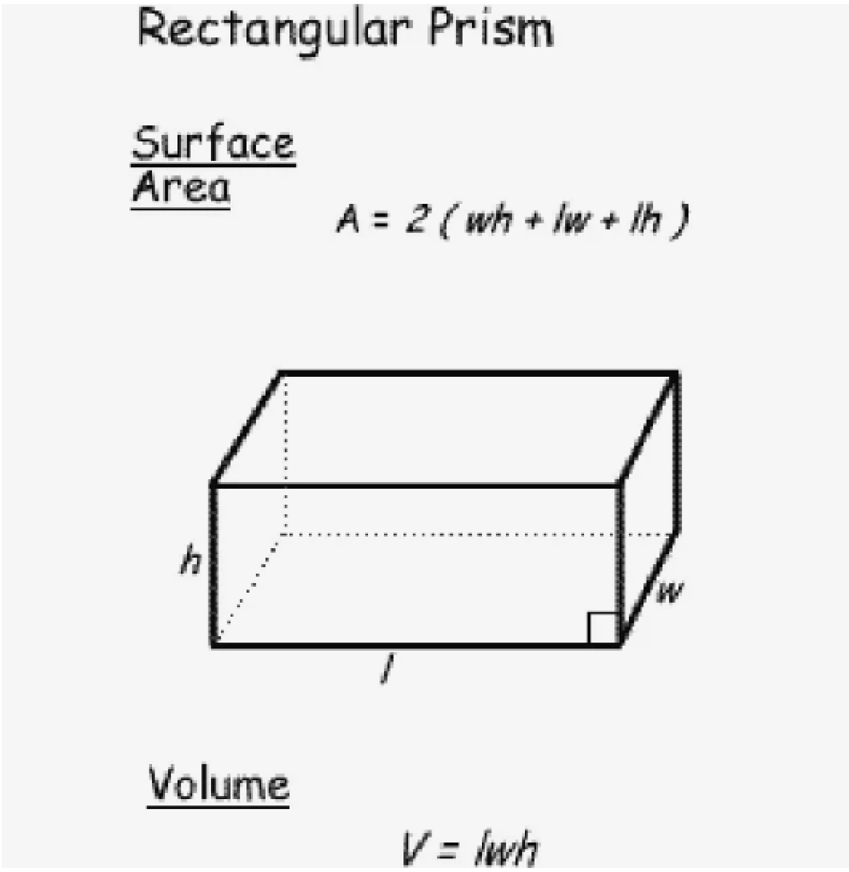


You will find that a cylinder is much easier to work with than a cone. This shape has a circular base and straight, parallel sides. This means that in order to find its surface area or volume, you only need the radius (**r**) and height (**h**).

However, you must also factor in that there is both a top and a bottom, which is why the radius must be multiplied by two for the surface area.

- **Surface Area =** $2\pi r^2 + 2\pi rh$
- **Volume =** $\pi r^2 h$

04. Surface Area and Volume of a Rectangular Prism

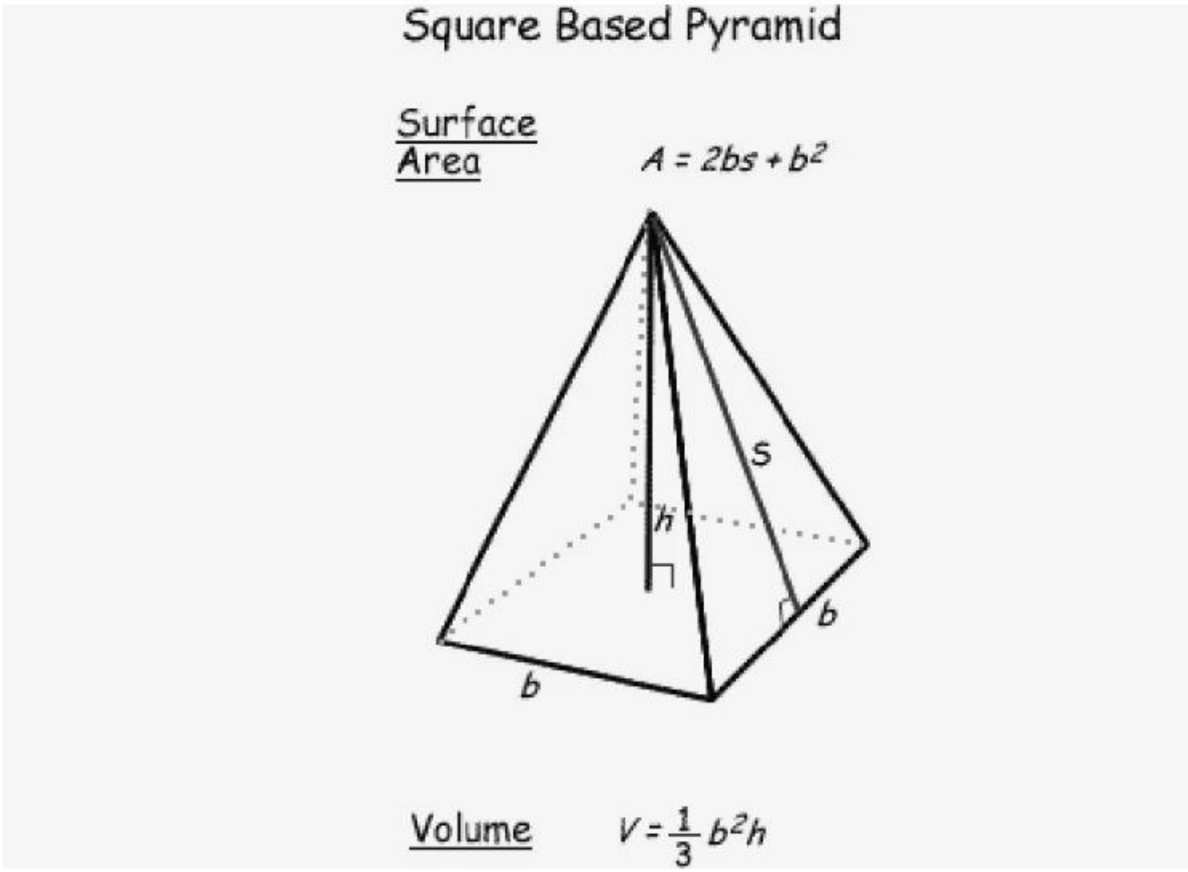


A rectangular in three dimensions becomes a rectangular prism (or a box). When all sides are of equal dimensions, it becomes a cube. Either way, finding the surface area and the volume require the same formulas.

For these, you will need to know the length (**l**), the height (**h**), and the width (**w**). With a cube, all three will be the same.

- **Surface Area = 2(lh) + 2(lw) + 2(wh)**
- **Volume = lhw**

05. Surface Area and Volume of a Pyramid



A pyramid with a square base and faces made of equilateral triangles is relatively easy to work with.

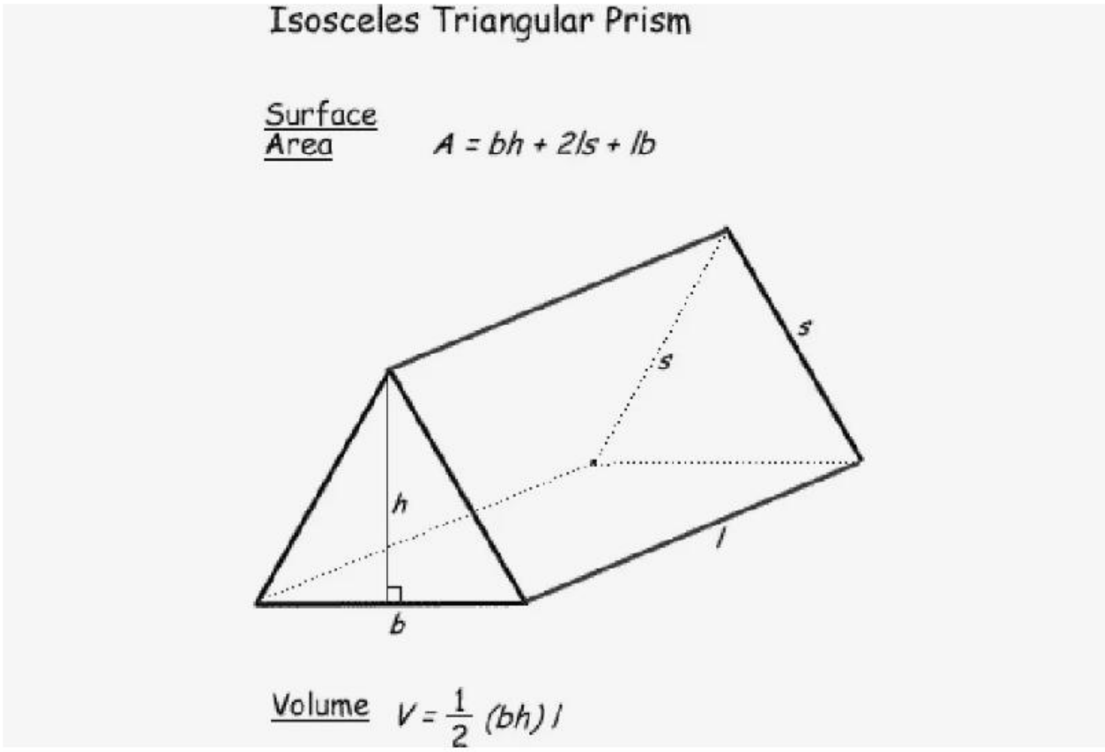
You will need to know the measurement for one length of the base (**b**). The height (**h**) is the distance from the base to the center point of the pyramid. The side (**s**) is the length of one face of the pyramid, from the base to the top point.

- **Surface Area = 2bs + b^2**
- **Volume = 1/3 b^2h**

Another way to calculate this is to use the perimeter (**P**) and the area (**A**) of the base shape. This can be used on a pyramid that has a rectangular rather than a square base.

- **Surface Area = (½ x P x s) + A**
- **Volume = 1/3 Ah**

06. Surface Area and Volume of a Prism



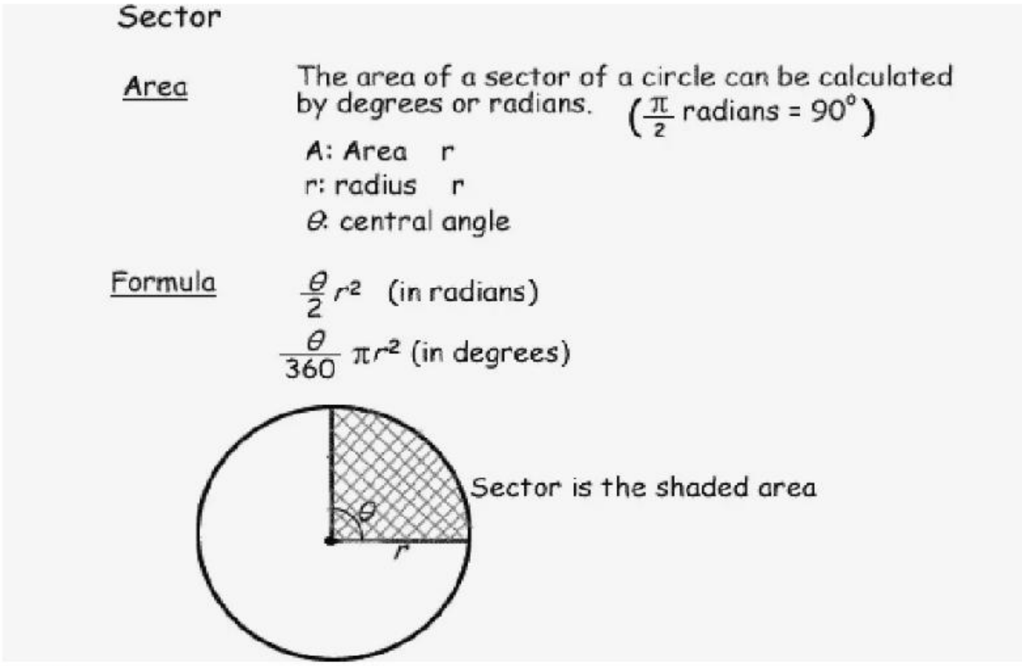
When you switch from a pyramid to an isosceles triangular prism, you must also factor in the length (**l**) of the shape. Remember the abbreviations for base (**b**), height (**h**), and side (**s**) because they are needed for these calculations.

- **Surface Area = bh + 2ls + lb**
- **Volume = 1/2 (bh)l**

Yet, a prism can be any stack of shapes. If you have to determine the area or volume of an odd prism, you can rely on the area (**A**) and the perimeter (**P**) of the base shape. Many times, this formula will use the height of the prism, or depth (**d**), rather than the length (**l**), though you may see either abbreviation.

- **Surface Area = 2A + Pd**
- **Volume = Ad**

07. Area of a Circle Sector

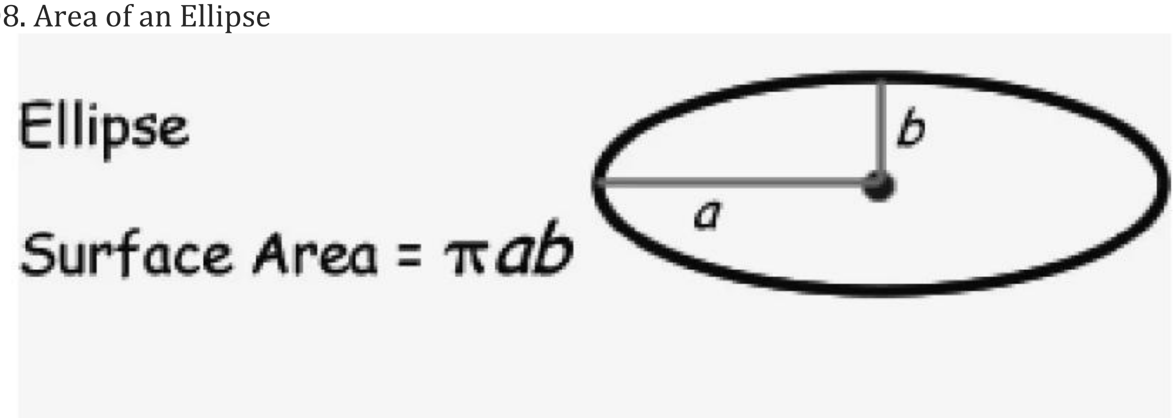


D. Russell

The area of a sector of a circle can be calculated by degrees (or radians as is used more often in calculus). For this, you will need the radius (**r**), pi (**π**), and the central angle (**θ**).

- **Area = θ/2 r**^{2} (in radians)
- **Area = θ/360 πr**^{2} (in degrees)

08. Area of an Ellipse



An ellipse is also called an oval and it is, essentially, an elongated circle. The distances from the center point to the side are not constant, which does make the formula for finding its area a little tricky.

To use this formula, you must know:

- Semiminor Axis (**a**): The shortest distance between the center point and the edge.
- Semimajor Axis (**b**): The longest distance between the center point and the edge.

The sum of these two points does remain constant. That is why we can use the following formula to calculate the area of any ellipse.

- **Area = πab**

On occasion, you may see this formula written with r_1 (radius 1 or semiminor axis) and r_2 (radius 2 or semimajor axis) rather than **a** and **b**.

- **Area = $\pi r_1 r_2$**

09. Area and Perimeter of a Triangle

The triangle is one of the simplest shapes and calculating the perimeter of this three-sided form is rather easy. You will need to know the lengths of all three sides (**a, b, c**) to measure the full perimeter.

- **Perimeter = a + b + c**

To find out the triangle's area, you will need only the length of the base (**b**) and the height (**h**), which is measured from the base to the peak of the triangle. This formula works for any triangle, no matter if the sides are equal or not.

- **Area = 1/2 bh**

10. Area and Circumference of a Circle

Similar to a sphere, you will need to know the radius (**r**) of a circle to find out its diameter (**d**) and circumference (**c**). Keep in mind that a circle is an ellipse that has an equal distance from the center point to every side (the radius), so it does not matter where on the edge you measure to.

- **Diameter (d) = 2r**
- **Circumference (c) = πd or $2\pi r$**

These two measurements are used in a formula to calculate the circle's area. It's also important to remember that the ratio between a circle's circumference and its diameter is equal to pi (**π**).

- **Area = πr^2**
-

11. Area and Perimeter of a Parallelogram

The parallelogram has two sets of opposite sides that run parallel to one another. The shape is a quadrangle, so it has four sides: two sides of one length (**a**) and two sides of another length (**b**).

To find out the perimeter of any parallelogram, use this simple formula:

- **Perimeter = 2a + 2b**

When you need to find the area of a parallelogram, you will need the height (**h**). This is the distance between two parallel sides. The base (**b**) is also required and this is the length of one of the sides.

- **Area = b x h**

Keep in mind that the **b** in the area formula is not the same as the **b** in the perimeter formula. You can use any of the sides—which were paired as **a** and **b** when calculating perimeter—though most often we use a side that is perpendicular to the height.

12. Area and Perimeter of a Rectangle

The rectangle is also a quadrangle. Unlike the parallelogram, the interior angles are always equal to 90 degrees. Also, the sides opposite one another will always measure the same length.

To use the formulas for perimeter and area, you will need to measure the rectangle's length (**l**) and its width (**w**).

- **Perimeter = 2h + 2w**
- **Area = h x w**

13. Area and Perimeter of a Square

The square is even easier than the rectangle because it is a rectangle with four equal sides. That means you only need to know the length of one side (**s**) in order to find its perimeter and area.

- **Perimeter = 4s**
- **Area =** s^2

14. Area and Perimeter of a Trapezoid

The trapezoid is a quadrangle that can look like a challenge, but it's actually quite easy. For this shape, only two sides are parallel to one another, though all four sides can be of different lengths. This means that you will need to know the length of each side (**a,** b_1**,** b_2**, c**) to find a trapezoid's perimeter.

- **Perimeter = a + b_1 + b_2 + c**

To find the area of a trapezoid, you will also need the height (**h**). This is the distance between the two parallel sides.

- **Area = 1/2 (b_1 + b_2) x h**

15. Area and Perimeter of a Hexagon

A six-sided polygon with equal sides is a regular hexagon. The length of each side is equal to the radius (**r**). While it may seem like a complicated shape, calculating the perimeter is a simple matter of multiplying the radius by the six sides.

- **Perimeter = 6r**

Figuring out the area of a hexagon is a little more difficult and you will have to memorize this formula:

- **Area = $(3\sqrt{3}/2)r^2$**

16. Area and Perimeter of an Octagon

A regular octagon is similar to a hexagon, though this polygon has eight equal sides. To find the perimeter and area of this shape, you will need the length of one side (**a**).

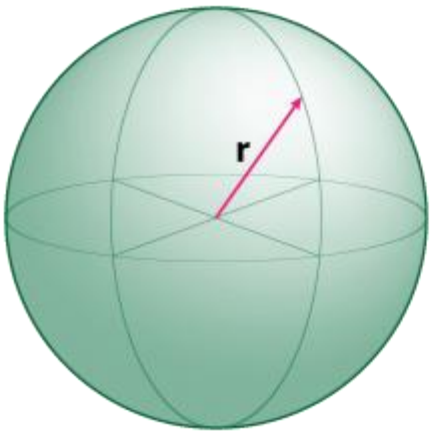
- **Perimeter = 8a**
- **Area = $(2 + 2\sqrt{2})a^2$**

Surface Area Formulas and Volume Formulas of 3D Shapes

This entry was posted on June 25, 2017 by Todd Helmenstine (updated on January 18, 2019)

Surface area formulas and volume formulas appear time and again in calculations and homework problems. Pressure is a force per area and density is mass per volume. These are just two simple types of calculations that involve these formulas. This is a short list of common geometric shapes and their surface area formulas and volume formulas.

Sphere Surface Area Formula and Sphere Volume Formula

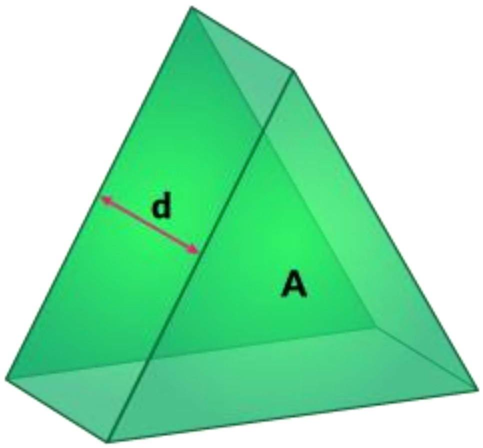


A sphere is a solid figure where every point on the surface is equidistant from the center of the sphere. This distance is the radius, r, of the sphere.

Surface area = $4\pi r^2$

Volume = $\frac{4}{3}\pi r^3$

Prism Surface Area Formula and Prism Volume Formula



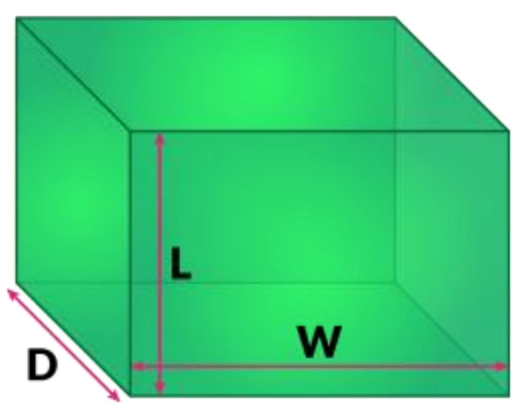
A prism is a geometric shape consisting of a stack of identical base shapes stacked on top of each other to a depth d. This prism is a prism formed by a stack of triangles.

Surface Area of a Prism = 2 × (Area of the base shape) + (Perimeter of base shape) × (d)

Volume of a Prism = (Area of base shape) × d

To find the area and perimeter of the base shape, check out Area Formulas and Perimeter Formulas.

Box Surface Area Formula and Box Volume Formula



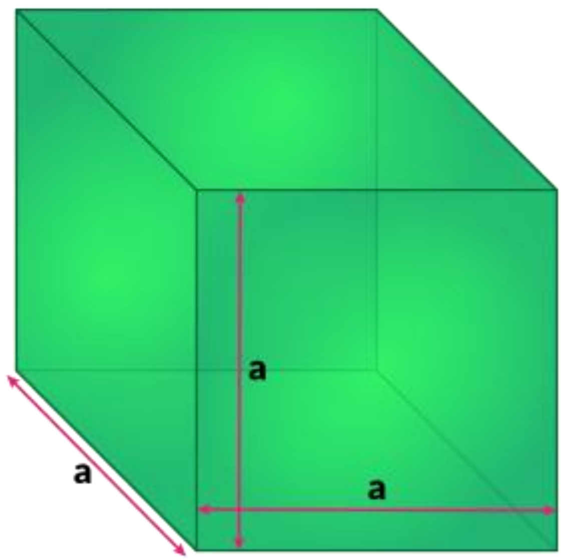
A box can be thought of a stack of rectangles L long and W wide piled on top of each other to a depth of D.

Surface Area of a Box = Sum of the areas of each face of the box, or

Surface Area of a Box = 2(L × W) + 2(L × D) + 2(W × D)

Volume of a Box = L × W × D

Cube Surface Area Formula and Cube Volume Formula

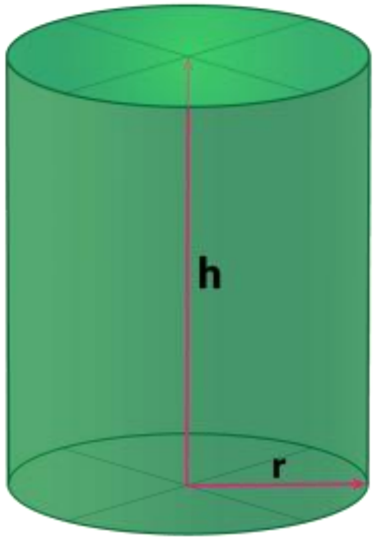


A cube is a special case box where all the sides are the same length.

Surface Area of a Cube = 6a^2

Volume of a Cube = a^3

Cylinder Surface Area Formula and Cylinder Volume Formula

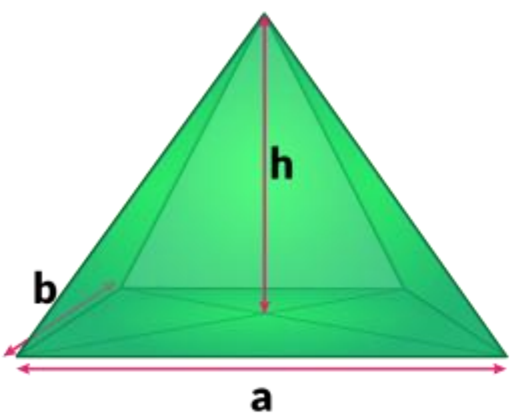


A cylinder is a prism where the base shape is a circle.

Surface Area of a Cylinder = $2\pi r^2 + 2\pi rh$

Volume of a Cylinder = $\pi r^2 h$

Pyramid Surface Area Formula and Pyramid Volume Formula



A pyramid is a solid shape consisting of a polygon base and triangular faces meeting at a common point above the base. The pyramid shown here is a rectangular pyramid. There are two important measurements needed to calculate surface area and volume of a pyramid. The first is the height of the pyramid (h). This is the distance from the base to the point where the triangular faces meet. The second is the height of the individual face triangles (s).

Surface Area of a Pyramid = (sum of the areas of each face) + (area of the base)

Volume of a Pyramid = $\frac{1}{3}$ A × h

For pyramids with identical face triangles

Surface Area of a Pyramid = ($\frac{1}{2}$ × Perimeter of base shape × s) + (Area of base shape)

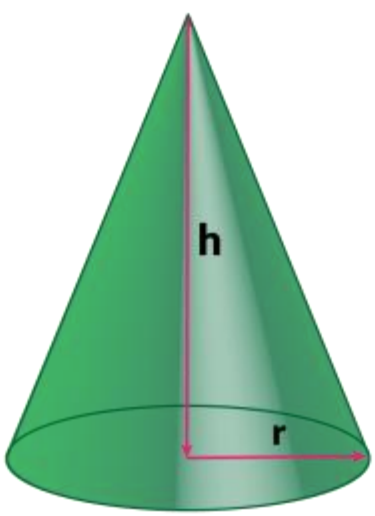
Volume of a Pyramid = $\frac{1}{3}$ A × h

If the base of the pyramid is a square (a = b), then

Surface area of a Square Pyramid = $a^2 + \sqrt{3}(a^2)$

Volume of a Square Pyramid = $\sqrt{5}(a^3/6)$

Surface Area Formula of a Cone and Volume Formula of a Cone



A cone is a pyramid with a circular base with radius r and height h. The side length s can be found using the Pythagorean Theorem.

$s^2 = r^2 + h^2$
or
$s = \sqrt{(r^2 + h^2)}$

Surface Area of a Cone = $\pi r^2 + \pi rs$

Volume of a Cone = $\frac{1}{3}(\pi r^2 h)$

VIDEOS

REFERENCES:

PTBB general math class 10 chapter 9

UNIT 34

Distance Formula

SLO'S:

- Explain and define coordinate geometry.
- Derive distance formula to calculate distance between two points given in Cartesian plane.
- Apply distance formula to find distance between two given points.

Introduction to Coordinate Geometry

A system of geometry where the position of points on the plane is described using an ordered pair of numbers.

Recall that a plane is a flat surface that goes on forever in both directions. If we were to place a point on the plane, coordinate geometry gives us a way to describe exactly where it is by using two numbers.

What are coordinates?

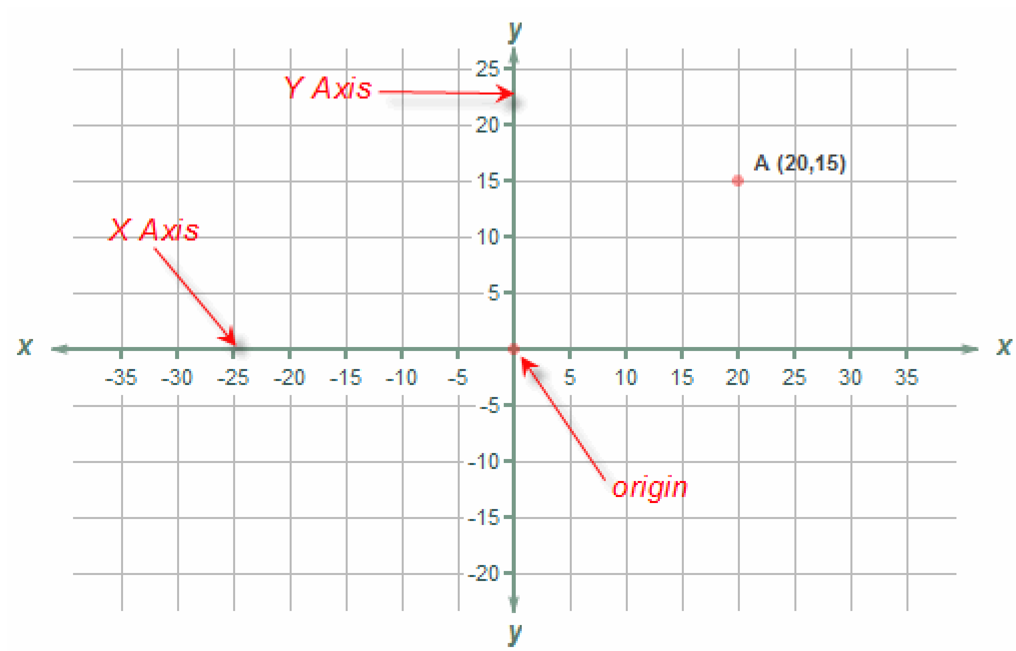
	A	B	C	D	E	F
1						
2						
3				**X**		
4						
5						
6						

To introduce the idea, consider the grid above. The columns of the grid are lettered A,B,C etc. The rows are numbered 1,2,3etc from the top. We can see that the **X** is in box D3; that is, column D, row 3.

D and 3 are called the *coordinates* of the box. It has two parts: the row and the column. There are many boxes in each row and many boxes in each column. But by having both we can find one single box, where the row and column intersect.

The Coordinate Plane

In coordinate geometry, points are placed on the "coordinate plane" as shown below. It has two scales - one running across the plane called the "x axis" and another a right angles to it called the y axis. (These can be thought of as similar to the column and row in the paragraph above.) The point where the axes cross is called the **origin** and is where both x and y are zero.



On the x-axis, values to the right are positive and those to the left are negative.
On the y-axis, values above the origin are positive and those below are negative.

A point's location on the plane is given by two numbers,the first tells where it is on the x-axis and the second which tells where it is on the y-axis. Together, they define a single, unique position on the plane. So in the diagram above, the point A has an x value of 20 and a y value of 15. These are the coordinates of the point A, sometimes referred to as its "rectangular coordinates". **Note** that the order is important; the x coordinate is always the first one of the pair.

For a more in-depth explanation of the coordinate plane see The Coordinate Plane.
For more on the coordinates of a point see Coordinates of a Point

Things you can do in Coordinate Geometry

If you know the coordinates of a group of points you can:

- Determine the distance between them
- Find the midpoint, slope and equation of a line segment
- Determine if lines are parallel or perpendicular
- Find the area and perimeter of a polygon defined by the points
- Transform a shape by moving, rotating and reflecting it.
- Define the equations of curves, circles and ellipses.

Information on all these and more can be found in the pages listed below.

History

The method of describing the location of points in this way was proposed by the French mathematician René Descartes (1596 - 1650). (Pronounced "day CART"). He proposed further that curves and lines could be described by equations using this technique, thus being the first to link algebra and geometry. In honor of his work, the coordinates of a point are often referred to as its Cartesian coordinates, and the coordinate plane as the Cartesian Coordinate Plane.

Coordinate Geometry

Do you remember what a plane is? A plane is any flat surface which can go on infinitely in both of the directions. Now, if there is a point on a plane, you can easily locate that point with the help of coordinate geometry. Using the two numbers of the coordinate geometry, a location of any point on the plane can be found. Let us know more!

A coordinate geometry is a branch of geometry where the position of the points on the plane is defined with the help of an ordered pair of numbers also known as coordinates.

What are Coordinates?

Now, to help you understand the coordinates, take a look at the figure below.

	A	B	C	D	E	F
1						
2						
3				X		
4						
5						
6						

Now, consider the grid on the right. The columns of the grid are labeled as A, B, C, D, E, F, etc. On the other hand, the rows are numbered as 1, 2, 3, 4, 5, 6, and so on. You can see that the letter X is located in the box D3 i.e. column D and row 3. Here, D and 3 are the coordinates of this box.

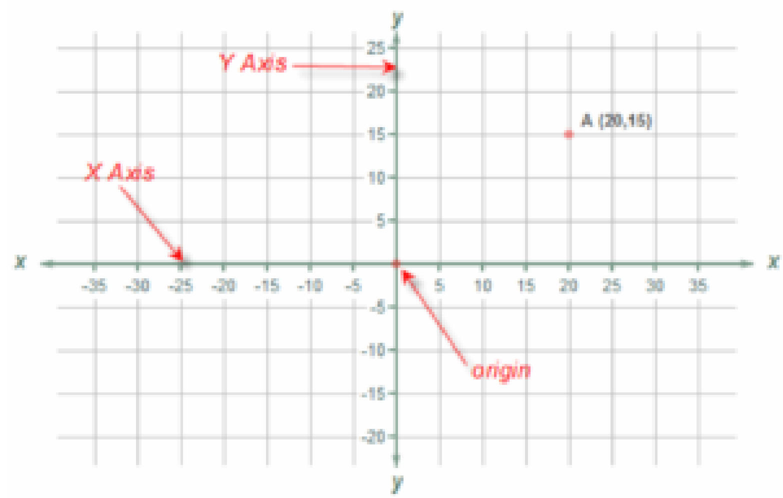
The box has two parts – one is the row and the other is the column. You need to understand that there are several boxes in every row and several boxes in every column. So, when you have both of them, you can find one single box that is the point where the rows and the columns intersect each other.

Browse more Topics Under Coordinate Geometry

- Coordinate Geometry
- Areas of Triangles and Quadrilaterals
- Distance Formula
- Section Formula

The Coordinate Plane

In the coordinate geometry, all the points are located on the coordinate plane. Take a look at the figure below.



The figure above has two scales – One is the X-axis which is running across the plane and the other one is the y-axis which is at the right angles to the X-axis. This is similar to the concept of the rows and columns that we discussed in the first part above.

Understanding the Concept of Coordinates

- The point of intersection of the x and the y-axis is known as the origin. At this point, both x and y are 0.
- The values on the right-hand side of the x-axis are positive and the values on the left-hand side of the x-axis are negative.
- Similarly, on the y-axis, the values located above the origin are positive and the values located below the origin are negative.
- When you have to locate a point on the plane, it is determined by a set of two numbers. So, first, you have to write about its location on the x-axis followed by its location on the y-axis. Together, the two will determine a single and unique position on the plane.

So, in the figure above, the point A has a value 20 on the x-axis and value 15 on the y-axis. These are also the coordinates of the point A. Often these points are also regarded as the "rectangular coordinates". Please note: The order of the points on the plane is crucial. You have to write the x coordinate ahead of the y coordinate.

Things That Have Been Made Possible By Coordinate Geometry

If you know the coordinates of a group of points, you can do the following:

a. Determine the distance between these points.

b. Find the equation, midpoint, and slope of the line segment.

c. Determine if the given lines are perpendicular or parallel.

d. Find the perimeter and the area of the polygon formed by the points on the plane.

e. Transform the shape by reflecting, moving and rotating it.

f. Define the equations of ellipses, curves, and circles.

Question For You

Q. What is the name of horizontal and vertical lines that are drawn to find out the position of any point in the Cartesian plane?

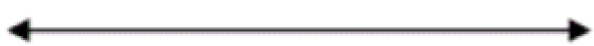
A: The name of horizontal and vertical lines that are drawn to find out the position of any point in the Cartesian plane are determined by x-axis and y-axis respectively.

Lines

In our childhood, our first experience with a pencil would surely have been associated with drawing random lines. But, do you know lines are the most vital element of ancient geometry. Moreover, this geometric figure has led to the development of several modern day theories which we are studying at present. Let us try to relate to this concept and form an understanding of the significance of line in mathematics.

Definition of Line

In a precise manner, a line doesn't hold a beginning or end point. You can imagine it continuing infinitely in both directions. We can demonstrate it by little arrows trailing at both ends.



Browse more Topics under Basic Geometrical Ideas

- Basic Geometrical Shapes
- Circle
- Curves
- Polygons and Angles
- Triangles and Quadrilaterals

Line Segment

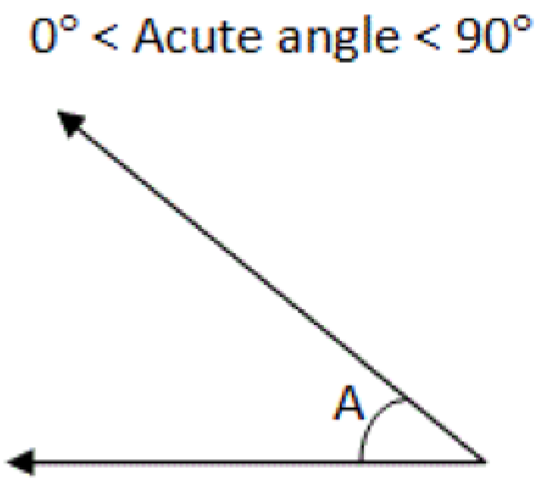
When two points are linked with a straight line, this is when we get a line segment. This below-mentioned line segment is AB.

Ray

A ray initiates from a point and lasts off to infinity. This can be represented by drawing an arrow symbol at one end of the ray. Sunrays can be a perfect example that initiates from the sun and travels indefinitely.

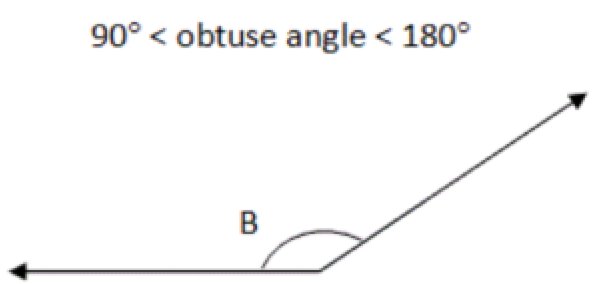
Acute Angle

From the figure drawn below, the angle falling between 0° and 90° is termed as an acute angle.



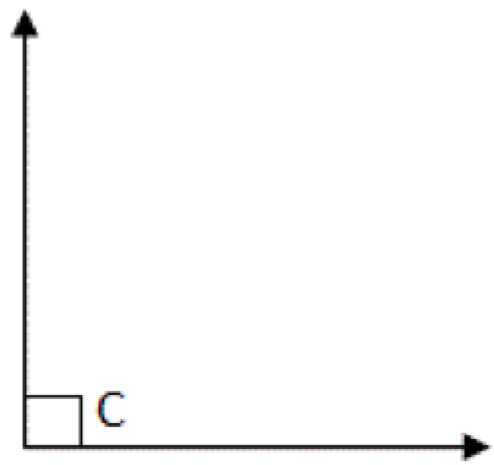
Obtuse Angle

An angle falling between 90° and 180° is an **obtuse angle**. From the figure, ∠B is an obtuse angle.



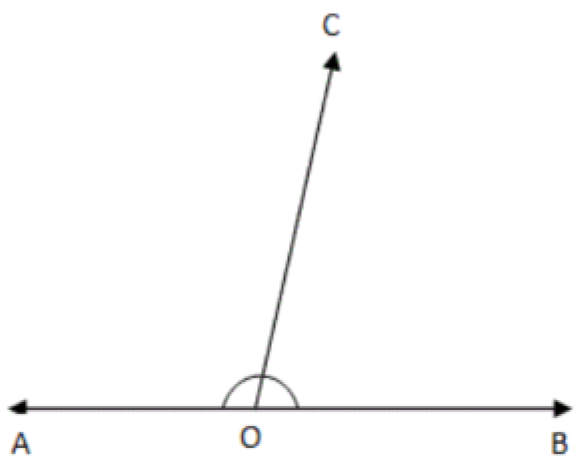
Right Angle

An angle that is 90° is called as a **Right angle**. In the figure, ∠C represents a right angle.



Supplementary Angles

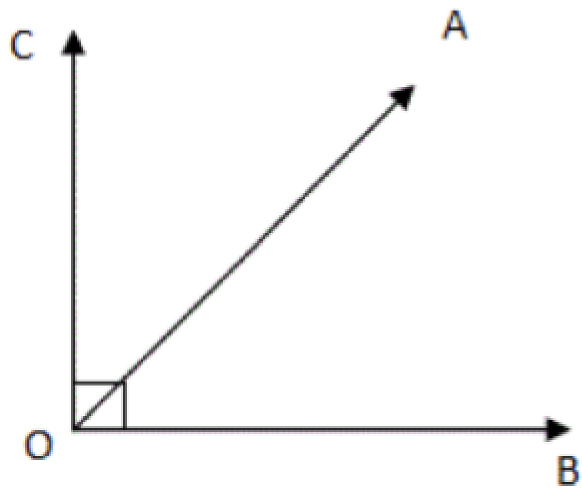
Based on the figure, ∠AOC + ∠COB = ∠AOB = 180° If the addition of two angles is 180°; in this case, the angles are termed as **supplementary angles**.



Further, it should be noted that two right angles would always supplement each other. Also, the pair of adjacent angles which when added form a straight angle is termed as a linear pair.

Complementary Angles

Based on the figure, ∠COA + ∠AOB = 90°.



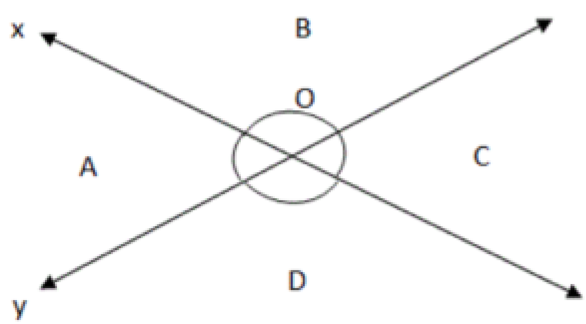
Hence, if the sum of two angles is 90°; in this case, the two angles are known as complementary angles.

Adjacent Angles

The angles which hold a common arm, as well as a common vertex, are termed as adjacent angles. Therefore, referring the above figure ∠BOA and ∠AOC are known as **adjacent angles**. OA is the common arm, with common vertex 'O'.

Vertically Opposite Angles

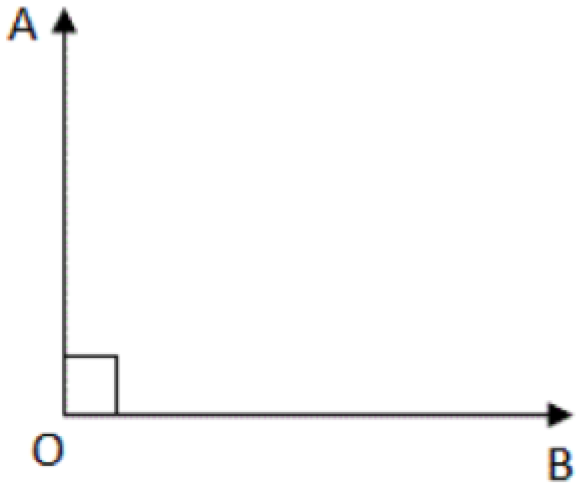
Whenever two **lines** intersect, the formation of angles is opposite to each other specifically at the point of intersection (vertex). These are termed as vertically opposite angles.



From the above figure above, x and y are seen as the intersecting lines. ∠A and ∠C form one pair of vertically opposite angles, whereas, ∠B and ∠D is the other pair of vertically opposite angles.

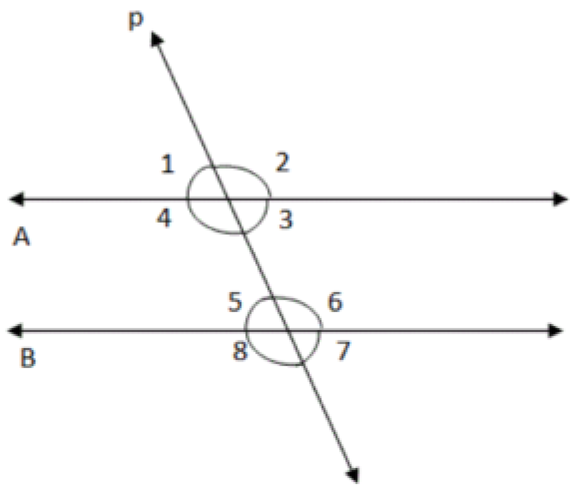
Perpendicular Lines

Whenever there is a right angle in the middle of two lines, then the lines are known to be perpendicular to each other.



From the figure, the lines OA and OB are termed as perpendicular to each other.

Parallel Lines



Referring to the figure, A and B are the two parallel lines, which are intersected by a line p. Here, the line p is known as a transversal, which meets two or more lines at distinct points.

Transversal Intersecting Two Parallel Lines

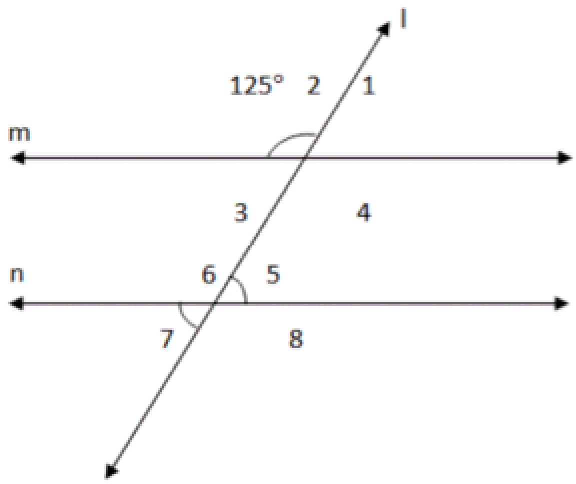
Under this condition, one must remember that:

- The corresponding angles tend to be equal.
- Vertically opposite angles become equal.
- Alternate exterior angles are equal.
- Alternate interior angles are equal.

- Pair of interior angles falling on the same side of the transversal is supplementary.

Question For You

Q. Suppose lines m and n tend to be parallel, then define the angles ∠5 and ∠7.



Solution: It is mentioned that, ∠2 = 125°
∠2 = ∠4 since vertically opposite angles. Hence, ∠4 = 125°
Now, observe that ∠4 is one of the interior angles falling on the same side of the transversal.
Thus, ∠4 + ∠5 = 180°
125 + ∠5 = 180 → ∠5 = 180 – 125 = 55°
∠5 = ∠7 because vertically opposite angles.
Therefore, ∠5 = ∠7 = 55°.

Polygons and Angles

Learning about geometric shapes and figures is very much important in order to build a relationship with the various structures present around us. Have you heard about a polygon? What exactly do you infer? From real-life objects, a STOP sign or a starfish, both are forms of a polygon. In this lesson, we'll learn what is an angle and what is a polygon.

Definition of Polygon

In simple mathematics, a polygon can be any 2-dimensional shape that is formed with straight lines. Be it quadrilaterals, triangles and pentagons, these are all perfect examples of polygons. The interesting aspect is that the name of a polygon highlights the number of sides it possesses.

For example, a triangle has three sides, and a quadrilateral has four sides. So, any shape that can be drawn by connecting three straight lines is called a triangle, and any shape that can be drawn by connecting four straight lines is called a quadrilateral.

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Types of Polygons

It should be known that polygons are categorized as different types depending on the number of sides together with the extent of the angles. Some of the prime categories of polygons include regular polygons, irregular polygons, concave polygons, convex polygons, quadrilateral polygons, pentagon polygons and so on.

Some of the most well-known polygons are triangles, squares, rectangles, parallelograms, pentagons, rhombuses, hexagons etc.

Regular polygon

Considering a regular polygon, it is noted that all sides of the polygon tend to be equal. Furthermore, all the interior angles remain equivalent.

Irregular polygon

These are those polygons that aren't regular. Be it the sides or the angles, nothing is equal as compared to a regular polygon.

Concave polygon

A concave polygon is that under which at least one angle is recorded more than 180 degrees. Also, the vertices of a concave polygon are both inwards and outwards.

Convex polygon

The measure of interior angle stays less than 180 degrees for a convex polygon. Such a polygon is known to be the exact opposite of a concave polygon. Moreover, the vertices associated to a convex polygon are always outwards.

Quadrilateral polygon

Four-sided polygon or quadrilateral polygon is quite common. There are different versions of a quadrilateral polygon such as square, parallelogram and rectangle.

Pentagon polygon

Pentagon polygons are six-sided polygons. It is important to note that, the five sides of the polygon stay equal in length. A regular pentagon is a prime type of pentagon polygon.

Formulae Related to Polygon

(N = count of sides and S = distance from center to a corner)

Regular polygon Area = (1/2) N sin(360°/N) S2

The number of diagonals = 1/2 N(N-3)

Summation of the interior angles of the polygon = (N – 2) x 180°

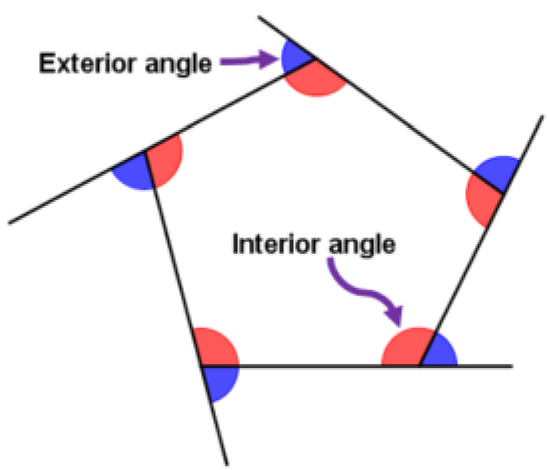
The count of triangles (while drawing all the diagonals through a single vertex) in a polygon = (N – 2)

What Is An Angle?

The study of angles is very important whenever we are trying to understand polygons and their properties. To be precise, when two rays hold a common endpoint, in this case, the two rays together form an angle. Therefore, an angle is formed by two rays initiating from a shared endpoint. These two rays creating it are termed as the sides or arms of the angle. For representing an angle the symbol "∠" is used in geometry.

Angles of Polygons

One must keep in mind that all polygons possess internal angles and external angles. In addition, a polygon's external angle can be termed as that which is extended on one side. Here are certain rules which are followed regarding angles of a polygon.



- Exterior Angle of a Polygon: All the Exterior Angles associated to a polygon add to form a sum 360°.

- Interior Angle of a Polygon: The Interior and exterior angle are evaluated through the same line, therefore, they add up to 180°.

That is, Interior Angle = 180° − Exterior Angle

Question For You

Q. What is the exact interior angle for a regular octagon?

Ans: Since a regular octagon possesses 8 sides.

Therefore, Exterior Angle = Total Degree of Polygon/Side= 360° / 8 = 45°

Hence, Interior Angle = 180° − 45° = 135°

VIDEOS

REFERENCES:

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UNIT 35

Collinear Points

SLO'S:

- Define collinear points. Distinguish between collinear and non- collinear points.
- Apply distance formula to show that three or more given points are collinear.
- Apply distance formula to show that the given three non-collinear points form are the followings:

  a) An equilateral triangle.

  b) An isosceles triangle.

  c) A right angled triangle.

Collinear points

Points that lie on the same line are called **collinear points**. If there is no line on which all of the points lie, then they are **non collinear points**.

What is the difference between collinear and non collinear?

In order for three or more points to be collinear, they must lie on the same line. Two points would always be collinear. Noncollinear are points that do not lie in the same line.

What is the difference between collinear and non collinear point?

Collinear points are three or more points lying on the same line. Non-collinear point are when less than three (not including three) points lie on a line.

When graphing the points can your tell whether or not they are collinear?

Yes. Calculate the ratio of the difference in y-coordinates and the difference in x-coordinates between pairs of points. If the ratio is the same, the points are collinear. If not, they are not. The only exception is if all the x-coordinates are he same and the ratio is not defined. In this case the points are also collinear - all on a vertical line

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What is similar between collinear and coplanar points?

because coplanar is coplanar and collinear is collinear!!

What is collinear and non collinear points?

Collinear points Points that lie on the same line are called collinear points. If there is no line on which all of the points lie, then they are non collinear points.

Mathematics-what is Pascal's theorem?

The three pairs of opposite sides of a hexagon inscribed in a conic intersect in collinear points

What does it mean for two points to be collinear?

"Collinear" means "on the same straight line". Two points are always collinear, because you can always draw a straight line between any two points. Three points may or may not be collinear.

What is a non collinear point?

Definition for collinear and non collinear Points that lie on the same line are called collinear points. If there is no line on which all of the points lie, then they are non collinear points.

What is a portion of a line that includes two points and all of the collinear points between the two points?

'Line Segment' is a portion of a line that includes two points and all of the collinear points between the hypothetical two points also 'Line Segment' because a line or line segment is a set of infinite points and the infinite points are collinear....

What is the symbol for collinear points?

The symbols for collinear points in Geometry are letters. Collinear points are defined as points which are located on the same line.

Is a line and point collinear?

If the point is not on the line, then no they are not collinear. But if that point is on the line, then they are collinear. Points on the same line are collinear. Points not on the same line are not collinear or non collinear.

What is non-collinear?
Non-collinear means that the points are not in a line. If only two points are given, they are always collinear.

How many points are collinear?
Points are collinear if they are on the same line.

What is the meaning of collinear points?
Collinear points are points on the same straight line.

What are some examples of collinear points?
Collinear points are points on a grid that lie on the same line. Non-collinear points do not sit on the same line.

What are collinear points?
Collinear points are points that lie on the same line. No collinear points do not lie on the same line. Any two points are always collinear, i.e. forming a line. Three or more points can be collinear along a single line. Collinear points lies on the same straight line.

How do you know if points are collinear?
Points are collinear if they lie on the same line.

Are 3 points collinear?
A set of 3 points will always be coplanar, but will only sometimes be collinear. Collinear points are always coplanar as well.

For a point to be between two other points the three points must be?
They must be collinear.

Is every set of three points collinear?
No, three points can be non collinear

Are collinear points also coplanar?
Yes, collinear points are also coplanar.

Are any two points collinear?
No. points are collinear only when they are on the same line

When you have three collinear points there is exactly one?
When you have three collinear points there is one gradient. I'm not sure what your question is specifically but when points are collinear they have the same gradient

Distance Formula

In Figure 1, *A* is (2, 2), *B* is (5, 2), and *C* is (5, 6).

Figure 1 Finding the distance from *A* to *C*.

To find *AB* or *BC*, only simple subtracting is necessary.

To find *AC*, though, simply subtracting is not sufficient. Triangle *ABC* is a right triangle with AC the hypotenuse. Therefore, by the *Pythagorean Theorem*,

If *A* is represented by the ordered pair (x_1, y_1) and *C* is represented by the ordered pair (x_2, y_2), then *AB* = ($x_2 - x_1$) and *BC* = ($y_2 - y_1$).

Then

This is stated as a theorem.

Theorem 101: If the coordinates of two points are (x_1, y_1) and (x_2, y_2), then the distance, *d*, between the two points is given by the following formula *(Distance Formula).*

Example 1: Use the *Distance Formula* to find the distance between the points with coordinates (−3, 4) and (5, 2).

Example 2: A triangle has vertices *A* (12, 5), *B* (5, 3), and *C* (12, 1). Show that the triangle is isosceles.

By the *Distance Formula,*

Because *AB* = *BC,* triangle *ABC* is isosceles.

REFERENCES:

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