ZIAUDDIN UNIVERSITY
EXAMINATION BOARD

Mathematics
IX-GEN
Teacher Resource

Topic: **Percentages**

Real Life Problems on Percentage

Real life problems on percentage will help us to solve different types of problems related to the real-life situations. To understand the procedures, follow step-by-step explanation so that you can solve any other similar type of percentage problems.

Solved real life problems on percentage:

1. Mike needs 30% to pass. If he scored 212 marks and falls short by 13 marks, what was the maximum marks he could have got?

Solution:

If Mike had scored 13 marks more, he could have scored 30%

Therefore, Mike required 212 + 13 = 225 marks

Let the maximum marks be m.

Then 30 % of m = 225

(30/100) × m = 225

m = (225 × 100)/30

m = 22500/30

m = 750

2. A number is increased by 40 % and then decreased by 40 %. Find the net increase or decrease per cent.

Solution:

Let the number be 100.

Increase in the number = 40 % = 40 % of 100

= (40/100 × 100)

= 40

Therefore, increased number = 100 + 40 = 140

This number is decreased by 40 %

Therefore, decrease in number = 40 % of 140

= (40/100 × 140)

= 5600/100

= 56

Therefore, new number = 140 - 56 = 84

Thus, net decreases = 100 - 84 = 16

Hence, net percentage decrease = (16/100 × 100) %

= (1600/100) %

= 16 %

3. Max scored 6 marks more than what he did in the previous examination in which he scored 30. Maria scored 30 marks more than she did in the previous examination in which she scored 60. Who showed less improvement?

Solution:

Max percentage improvement in the first exam = (6/30 × 100) %

= (600/30) %

= 20 %

Maria percentage improvement in the first exam = (30/60 × 100) %

= (3000/60) %

= 50 %

Hence, 20 % < 50 %

Therefore, Max showed less improvement.

II.Topic: Ratio

Concept of Ratio

In concept of ratio we will learn how a ratio is compared with two or more quantities of the same kind. It can be represented as a fraction.

A ratio is a comparison of two or more quantities of the same kind. It can be represented as a fraction.

Most of time, we compare things, number, etc. (say, **m** and **n**) by saying:

(i) **m** greater than **n**

(ii) **m** less than **n**

When we want to see how much more (**m** greater than **n**) or less (**m** less than **n**) one quantities is than the other, we find the difference of their magnitudes and such a comparison is known as the comparison by division.

(iii) **m** is double of **n**

(iv) **m** is one-fourth of **n**

If we want to see how many times more (**m** is double of **n**) or less (**m** is one-fourth of **n**) one quantities is than the other, we find the ratio or division of their magnitudes and such a comparison is known as the comparison by difference.

(v) **m**/**n** = 2/3

(vi) **n**/**m** = 5/7, etc.

The method of comparing two quantities (numbers, things, etc.) by dividing one quantity by the other is called a ratio.

Thus: (v) **m**/**n** = 2/3 represents the ratio between **m** and **n**.

(vi) **n**/**m** = 5/7 represents the ratio between **n** and **m**.

When we compare two quantities of the same kind of division, we say that we form a ratio of the two quantities.

Therefore, it is evident from the basic concept of ratio is that a ratio is a fraction that shows how many times a quantity is of another quantity of the same kind.

Definition of Ratio:

The relation between two quantities (both of them are same kind and in the same unit) obtain on dividing one quantity by the other, is called the ratio.

The symbol used for this purpose ":" and is put between the two quantities compared.

Therefore, the ratio between two quantities **m** and **n** ($n \neq 0$), both of them same kind and in the same unit, is **m/n** and often written as **m : n** (read as **m to n** or **m is to n**)

In the ratio m : n, the quantities (numbers) m and n are called the terms of the ratio. The first term (i.e. m) is called antecedent and the second term (i.e. is n) is called consequent.

Note: From the concept of ratio and its definition we come to know that when numerator and denominator of a fraction are divided or multiplied by the same non-zero numbers, the value of the fraction does not change. In this reason, the value of a ratio does not alter, if its antecedent and consequent are divided or multiplied by the same non-zero numbers.

For example, the ratio of 15 and 25 = 15 : 25 = 15/25

Now, multiply numerator (antecedent) and denominator (consequent) by 5

$15/25 = (15 \times 5)/(25 \times 5) = 75/125$

Therefore, 15/25 = 75/125

Again, divide numerator (antecedent) and denominator (consequent) by 5

$15/25 = (15 \div 5)/(25 \div 5) = 3/5$

Therefore, 15/25 = 3/5

Examples on ratio:

(i) The ratio of $ 2 to $ 3 = $ 2/$ 3 = 2/3 =2 : 3.

(ii) The ratio of 7 metres to 4 metres = 7 metres/4 metres = 7/4 = 7 : 4.

(iii) The ratio of 9 kg to 17 kg = 9 kg/17 kg= 9/17 = 9 : 17.

(iv) The ratio of 13 litres to 5 litres = 13 litres/5 litres = 13/5 = 13 : 5.

Topic: proportion and compound proportion

Proportion word problems

Problem

Chef Rita is cooking for a Sunday brunch. She knows that 22 pancakes can feed 8 people. She is wondering how many people (p) she can feed with 55 pancakes. She assumes each person eats the same quantity of pancakes.

How many people can Rita feed with 55 pancakes?

[ ] people

Yoku is putting on sunscreen. He uses 2 ml to cover 50 cm^2 of his skin. He wants to know how many milliliters of sunscreen (c) he needs to cover 325 cm^2 of his skin.

How many milliliters of sunscreen does Yoku need to cover 325 cm^2 of his skin?

[ ] ml

Kwesi is putting on sunscreen. He uses 3 ml to cover 45 cm^2 of his skin. He wants to know how many milliliters of sunscreen (g) he needs to cover 240 cm^2 of his skin. He assumes the relationship between milliliters of sunscreen and area is proportional.

How many milliliters of sunscreen does Kwesi need to cover 240 cm^2 of his skin?

[ ] ml

Teresa is maintaining a camp fire. She can keep the fire burning for 4 hours with 6 logs. She wants to know how many logs (y) she needs to keep the fire burning for 18 hours. She assumes all logs are the same.

How many logs does Teresa need to maintain the fire for 18 hours?

[ ] logs

Topic: Profit loss and discount and profit markup (interest)

> **Cost Price:** The amount paid to purchase an article or the price at which an article is made, is known as its cost price. The cost price is abbreviated as C.P..
>
> **Selling Price:** The price at which an article is sold, is known as its selling price. The selling price is abbreviated as S.P.
>
> **Profit:** If the selling price (S.P.) of an article is greater than the cost price (C.P.) , then the difference between the selling price and cost price is called profit.

Thus, If S.P. > C.P., then

Profit = S.P. – C.P.

⇒ S.P. = C.P. + Profit

⇒ C.P. = S.P. – Profit.

Example 1: A shopkeeper buys scientific calculators in bulk for Rs. 15 each. He sells them for Rs. 40 each. Calculate the profit on each calculator in rupees, and as a percentage of the cost price.

Solution: Given: cost price = Rs. 15, selling price = Rs. 40

Profit = selling price – cost price = Rs. 40 – 15 = Rs. 25

the profit as a percentage of the cost price:

Profit % = $\frac{\text{profit}}{\text{cost price}} \times 100\%$

$= \frac{25 \times 100}{15}\%$ = 166.7%

> **Loss:** If the selling price (S.P.) of an article is less than the cost price (C.P.) , then the difference between the cost price (C.P.) and the selling price (S.P.) is called loss.

Thus, if S.P. < C.P., then Loss = C.P. – S.P. ⇒ C.P. = S.P. + Loss ⇒ S.P. = C.P. – Loss [/av_textblock] [av_textblock size='' av-medium-font-size='' av-small-font-size='' av-mini-font-size='' font_color='' color='' id='' custom_class='ed-text-block-sm-textp' av_uid='av-2wpi6q4' admin_preview_bg=''] **Example 2: If the cost price of a book is Rs. 150 and selling price is 137.50, then calculate the loss and percentage loss on the book?**

Solution: Here, cost price = Rs. 150

and selling price = Rs. 137.50

∴Loss = Cost price – selling price

= Rs. (150 – 137.50) = Rs. 12.50

Now, Percentage Loss

$= \frac{\text{Loss} \times 100}{\text{Cost Price}} \%$

$= \frac{12.50 \times 100}{150} \% = 8.33\%$

When cost price and percentage profit are given, then selling price

$= \text{cost price}\left(\frac{100+\text{profit\%}}{100}\right)$

Example 3: A chair was purchased for Rs. 470 and sold at a profit of 10%. Find the selling price.

Solution: Using the formula

Selling price = cost price $\left(\frac{100+\text{profit\%}}{100}\right)$

$= 470\left(\frac{100+10}{100}\right) = 470\times\frac{110}{100}$ = Rs. 517

When cost price and percentage loss are given, then

Selling price = cost price $\left(\frac{100-\text{Loss\%}}{100}\right)$

Example 4: A person bought a table for Rs. 420 and sold at the loss of 15% . Find the selling price of table?

Solution: Selling price = cost price $\left(\frac{100-\text{Loss\%}}{100}\right)$

= Rs. 420 $\left(\frac{100-15}{100}\right)=\frac{420\times 85}{100}$

= Rs. 357

> When selling price and percentage profit are given, then
>
> Cost price = selling price $\left(\frac{100}{100 + \text{profit\%}}\right)$

Example 5: A Chair was sold for Rs. 517 at a profit of 10%. Find the cost price of the chair.

Solution: Here, selling price = Rs. 517 and profit = 10%

∴ Cost price = selling price $\left(\frac{100}{100 + \text{profit\%}}\right)$

$= 517\left(\frac{100}{100 + 10}\right)$

= 517 × $\frac{100}{110}$ = Rs. 470

> When selling price and percentage loss are given, then
>
> Cost price = selling price $\left(\frac{100}{100 - \text{Loss\%}}\right)$

Example 6: Ram sold a watch for Rs. 376 at a loss of 6%. Find the cost price of the watch.

Solution:

Cost price = selling price $\left(\frac{100}{100 - \text{Loss\%}}\right)$

= Rs. 376 × $\left(\frac{100}{100 - 6}\right)$

= Rs. 376 × $\frac{100}{94}$ = Rs. 400

> If two items are sold each at rupees R, one at a gain of x% and other at a loss of x %, there is always an overall loss given by $\frac{x^2}{100}$ % and the value of loss is given by $\frac{2x^2R}{(100^2 - x^2)}$.

In case the cost price of both the items is the same and percentage loss and gain are equal, then net loss or profit is zero. The difference between the two cases is that the cost price in the first case is not the same, and in the second case it is the same.

Example 7: Ram sells two Mobile phones for Rs. 1000 each, one at a profit of 10% and other at a loss of 10%. Find his gain or loss percentage.

Solution: Using the formula,

Loss % = $\left(\frac{x^2}{100}\right)\% = \left(\frac{10 \times 10}{100}\right)\% = 1\%$

> A dishonest shopkeeper claims to sell goods at cost price, but uses a lighter weight, then his Gain %
>
> $$= \left[\frac{100 \times \text{excess}}{(\text{original value} - \text{excess})}\right]$$

Example 8: A shopkeeper sells rice to a customer, using false weights and gains 100/8 % on his cost. What weight has he substituted for a kilogram?

Solution: Using the formula,

$$\text{Gain \%} = \left[\frac{100 \times \text{excess}}{(\text{original value} - \text{excess})}\right]$$

$$\Rightarrow \frac{100}{8} = \left[\frac{100 \times \text{excess}}{(1 - \text{excess})}\right]$$

From here, Excess = 0.111.. Kg, which is 111.11 grams

Weight used by shopkeeper = 1000 – 111.11 = 888.89 grams

Formulas to Remember

$$\text{Profit} = \frac{\text{C.P.} \times \text{Profit \%}}{100}$$

$$\text{Loss} = \frac{\text{C.P.} \times \text{Loss \%}}{100}$$

$$\text{S.P.} = \left(\frac{100 + \text{Profit\%}}{100}\right) \times \text{C.P.}$$

$$\text{S.P.} = \left(\frac{100 - \text{Loss\%}}{100}\right) \times \text{C.P.}$$

$$\text{C.P.} = \frac{100 \times \text{S.P.}}{100 + \text{Profit \%}}$$

$$\text{C.P.} = \frac{100 \times \text{S.P.}}{100 - \text{Loss \%}}$$

Goods passing through successive hands

> When there are two successive profits of a% and b%, then the resultant profit per cent is given by
>
> $$\left(a+b+\frac{ab}{100}\right)\%$$
>
> When there is a profit of a% and loss by b% in a transaction, then the resultant profit or loss per cent is given by $\left(a-b-\frac{ab}{100}\right)\%$
>
> according to the +ve or –ve sign respectively.
>
> When cost price and selling price are reduced by the same amount (A) and profit increases then cost price (C.P.)
>
> $$=\frac{[\text{Initial profit}\% + \text{Increase in profit}\%]\times A}{\text{Increase in profit}\%}$$

Example 9: A table is sold at a profit of 20%. If the cost price and selling price are Rs. 200 less, the profit would be 8% more. Find the cost price.

Solution: By direct method,

C.P. = $\text{Rs.}\frac{(20+8)\times 200}{8}$

= Rs. 28 × 25 = Rs. 700.

> If cost price of x articles is equal to the selling price of y articles, then profit/loss percentage
>
> $$=\frac{x-y}{y}\times 100\%$$

according to +ve or –ve sign respectively.

Example 10: If the C.P. of 15 tables be equal to the S.P. of 20 tables, find the loss per cent.

Solution: By direct method,

Profit/Loss % = $\frac{-5}{20}\times 100$

= –25% loss, since it is –ve.

Discount

> The reduction made on the 'marked price' of an article is called the discount. When no discount is given, 'selling price' is the same as 'marked price'.
>
> Discount = Marked price × Rate of discount.
>
> S.P. = M.P. – Discount.
>
> Discount % = $\frac{\text{Discount}}{\text{M.P.}}\times 100$

Example 11: How much % must be added to the cost price of goods so that a profit of 20% must be made after throwing off a discount of 10% from the marked price?

Solution: (c) Let C.P. = Rs. 100, then S.P. = Rs. 120

Also, Let marked price be Rs. x. Then, 90% of x = 120

$\Rightarrow x = \frac{120 \times 100}{90} = 133\frac{1}{3}$

∴ M.P. should be Rs. $133\frac{1}{3}$

or M.P. = $33\frac{1}{3}\%$ above C.P.

> Buy x get y free i.e., if x + y articles are sold at cost price of x articles, then the percentage discount
>
> = $\frac{y}{x+y}$ × 100.

Successive Discounts

In successive discounts, first discount is subtracted from the marked price to get net price after the first discount. Taking this price as the new marked price, the second discount is calculated and it is subtracted from it to get net price after the second discount. Continuing in this manner, we finally obtain the net selling price.

In case of successive discounts a% and b%, the effective discount is:

$$\left(a+b-\frac{ab}{100}\right)\%$$

Example 12: Find the single discount equivalent to successive discounts of 15% and 20%.

Solution: By direct formula,

Single discount

$$=\left(a+b-\frac{ab}{100}\right)\%$$

$$=\left(15+20-\frac{15\times 20}{100}\right)\%$$

= 32 %

> If the list price of an item is given and discounts d_1 and d_2 are given successively on it then, Final price = list price$\left(1-\frac{d_1}{100}\right)\left(1-\frac{d_2}{100}\right)$

Example 13: An article is listed at Rs. 65. A customer bought this article for Rs. 56.16 and got two successive discounts of which the first one is 10%. The other rate of discount of this scheme that was allowed by the shopkeeper was :

1. 3%
2. 4%
3. 6%
4. 2%

Solution: (2): Price of the article after first discount = 65 – 6.5 = Rs. 58.5

Therefore, the second discount

$$= \frac{58.5-56.16}{58.5}\times 100 = 4\%$$

Profit and Loss: Concept of Discounts and Marked Price Explained

In the first part of Profit and Loss series, we learnt the basic definitions and the meaning of Cost Price, Selling Price, Marked price etc. Let us revise the definition of Marked Price. As we saw earlier, traders are in the habit of marking their articles at a certain price above their costs. Then the discounts they offer are on this marked price, thereby they actually make sure that have already factored in the profit they want.

Topic: Radicals and Radicands

Laws of Exponents

/Indices

Logarithm

Laws of Logarithm

Application of logarithm

Understanding Logarithms and Roots

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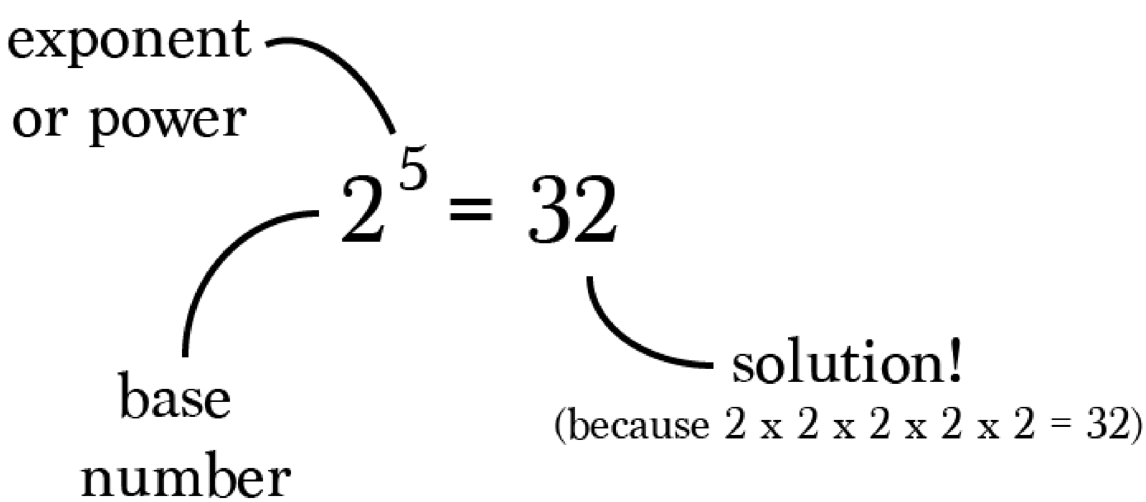
Oct 26, 2015 · 4 min read

Logs & roots — no, I'm not talking about trees. I'm talking about the mathematical kind. I bet you're thinking,

"Roots, okay.But logarithms? Isn't that an algebra 2 topic?!"

Yep, it is! But who says we can't learn it right now? Why save for later what can be learned today? *Carpe diem*, am I right? But first, let's do a quick review. Recall this diagram on exponents from lesson two?



The number we multiply with itself is called the **base.** The number of times we multiply it with itself is called the **power** or **exponent**.

Here are a few examples of exponents to refresh your memory.

$$9^2 = 9 \times 9 = 81$$

$$4^3 = 4 \times 4 \times 4 = 64$$

$$3^4 = 3 \times 3 \times 3 \times 3 = 81$$

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

We'd like to find an operation to get back the base number from the solution of an exponential equation. That's where roots come in.

Square Roots, Cube Roots and More

Suppose instead of finding the square of 9, which is 81, we wanted to find out what number multiplied with itself equals 81.

In other words, *what is the **square root** of 81?*

$$\sqrt[2]{81}$$

This equals 9 because nine squared is eighty-one.

$$9^2 = 81$$

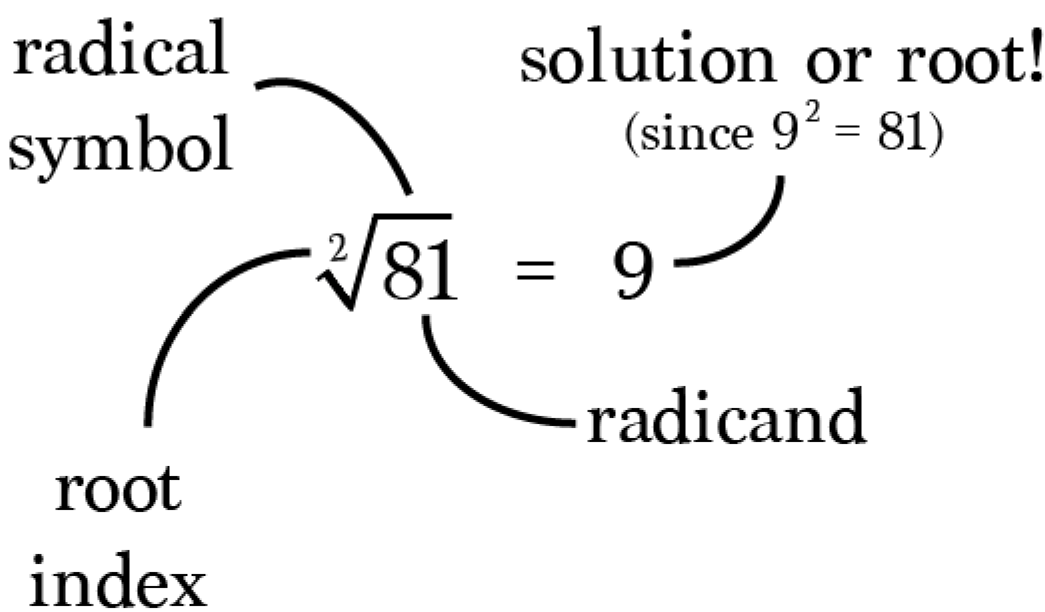
We can take the square root of any non-negative number, but only perfect square numbers yield whole number results. So familiarize yourself with those first. Here are a few to get you started:

$$\sqrt[2]{4} = 2 \qquad \sqrt[2]{9} = 3 \qquad \sqrt[2]{16} = 4$$

$$\sqrt[2]{25} = 5 \qquad \sqrt[2]{36} = 6 \qquad \sqrt[2]{49} = 7$$

$$\sqrt[2]{64} = 8 \qquad \sqrt[2]{81} = 9 \qquad \sqrt[2]{100} = 10$$

Now for some terminology.



The root index is *optional* on square roots. Square roots are often written:

$$\sqrt{64} = 8$$

The index is only necessary to distinguish between higher indexed roots, such as cube roots, fourth roots, fifth roots, etc.

Cube roots ask you to find the number that when multiplied with itself **three** times yields the radicand, like these:

$$\sqrt[3]{1} = 1 \text{ since } 1^3 = 1$$

$$\sqrt[3]{8} = 2 \text{ since } 2^3 = 8$$

$$\sqrt[3]{27} = 3 \text{ since } 3^3 = 27$$

$$\sqrt[3]{64} = 4 \text{ since } 4^3 = 64$$

$$\sqrt[3]{125} = 5 \text{ since } 5^3 = 125$$

Fourth roots ask you to find the number that when multiplied with itself **four** times yields the radicand.

$$\sqrt[4]{1} = 1 \text{ since } 1^4 = 1$$

$$\sqrt[4]{16} = 2 \text{ since } 2^4 = 16$$

$$\sqrt[4]{81} = 3 \text{ since } 3^4 = 81$$

Fifth roots ask you to find the number that when multiplied with itself **five** times yields the radicand.

$$\sqrt[5]{1} = 1 \text{ since } 1^5 = 1$$

$$\sqrt[5]{32} = 2 \text{ since } 2^5 = 32$$

Again, you can take *any root of any non-negative number* (and in certain cases of negative numbers as well), but for many numbers you'll need a calculator since the answers are irrational. The examples above are the "nice cases" that come up frequently!

Logarithms

What if we wanted to solve for the exponent in an exponential equation? In other words, we want to reverse the exponentiation. For example, what is the solution to this problem?

$$6^{?} = 36$$

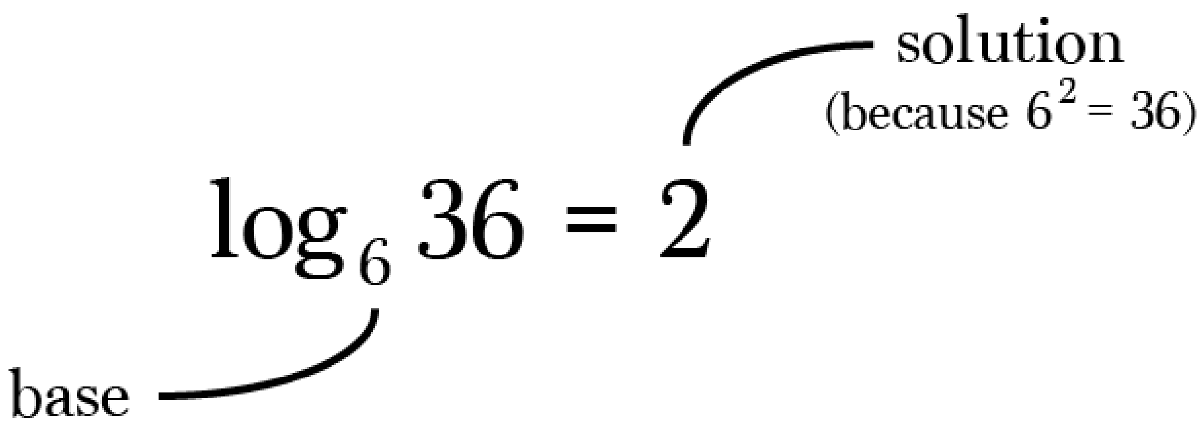
Since we've memorized the common powers and roots, we easily identify the solution as 2 since 6 to the power of 2 is 36.

Writing a question mark in the equation isn't formal mathematics, instead we'll write the above expression using **logarithm notation,** or **log** for short.

$$\log_6 36 = 2$$

Read: "*the log, base six, of thirty-six is 2*."

The terminology:



Another way to look at this is to ask,

"*How many sixes need to be multiplied together to get 36?*"

A **logarithm solves for the number of repeated multiplications.** Simple as that. Here are a few more examples.

$$\log_5 125 = 3 \quad \text{since } 5^3 = 125$$
$$\log_3 81 = 4 \quad \text{since } 3^4 = 81$$
$$\log_2 32 = 5 \quad \text{since } 2^5 = 32$$

Exponents and Radicals √(a + b) 10 Exponents are a very important part of algebra. An exponent is just a convenient way of writing repeated multiplications of the same number. Radicals involve the use of the radical sign (√). Sometimes these are called surds. If you learn the rules for exponents and radicals, then your enjoyment of mathematics will surely increase! 1.-Objetives Unit Objectives • To be able to illustrate the relationship between the radical and exponential forms of an equation. Supported by learning objectives 1, 3, and 5. • To demonstrate the ability to work with operations involving radical numbers. Supported by learning objectives 2, 4, 6, and 7. • To be able to solve equations involving radicals and to be able to justify the solutions. Supported by learning objectives 8 to 11.Learning Objectives 1.To evaluate powers with rational exponents. 2. To apply the laws of exponents to simplify expressions involving rational exponents. 3. To write exponential expressions in radical form. 4. To simplify square root and cube root expressions. 5. To write radical expressions in exponential form. 6. To add, subtract, multiply and divide square root and cube root expressions. 7. To rationalize monomial and binomial denominators in radical expressions. 8. To solve and verify the solutions of quadratic equations by factoring, completing the trinomial square, and using the quadratic formula. 9. To solve and verify equations involving absolute value. 10. To solve radical equations with two unlike radicands. 11. To solve word problems involving radical equations. 2.-Contents 2.1. Simplifying Expressions with Integral Exponents Laws of Exponents Integral Exponents Integral here means "integer". So the exponent (or power) is an integer. [That is, either a negative whole number, 0, or a positive whole number.] Definition: am means "multiply a by itself m times". That is: am = a × a × a × a × a × ... × a [We do the multiplication m times.] Note: This definition only really holds for m> 0, since it doesn't make a lot of sense if m is negative. (You can't multiply something by itself -3 times! And how does multiplying something by itself 0 times give 1?) In such cases we have to rely on patterns and conventions to define what is going on. See below for zero and negative exponents. Examples (1) y 5 = y × y × y × y × y (2) 24 = 2 × 2 ×

2 × 2 = 16. Multiplying Expressions with the Same Base Definition: am × an = am+n Let's see how this works with an example. Example b5 × b3 = (b × b × b × b × b) × (b × b × b) = b8 (that is, b5+3) Dividing Expressions with the Same Base Definition: (Of course, a ≠ 0) It may be easiest to see how this one works with an example. Example We cancel 2 of the b's from the numerator and the two b's from the denominator of the fraction. The result is equivalent to b7 − 2. Repeated Multiplication of a Number Raised to a Power (am) n = (am) × (am) × (am) × ... × (am) [We multiply n times]. = amn So we write: Definition: (am) n = amn Example (p 3) 2 = p 3 × p 3 = (p × p × p) × (p × p × p) = p 6 A Product Raised to an Integral Power Definition: (ab) n = an bn Example (5q) 3 = 53 q 3 = 125q 3 A Fraction Raised to an Integral Power Definition: Example Zero Exponents Definition: a 0 = 1 (a ≠ 0) Example 70 = 1 Note: a 0 = 1 is a convention, that is, we agree that raising any number to the power 0 is 1. We cannot multiply a number by itself zero times. In the case of zero raised to the power 0 (00), mathematicians have been debating this for hundreds of years. It is most commonly regarded as having value 1, but is not so in all places where it occurs. That's why we write a ≠ 0. Negative Exponents Definition: (Once again, a ≠ 0) In this exponent rule, a cannot equal 0 because you cannot have 0 on the bottom of a fraction. Example Explanations Observe the following decreasing pattern: 34 = 81 33 = 27 32 = 9 31 = 3 For each step, we are dividing by 3. Now, continuing beyond 31 and dividing by 3 each times gives us: Summary - Laws of Exponents [Note: These laws mostly apply if we have fractional exponents, which we meet in the next section, Fractional Exponents.] Let's try some mixed examples where we have integral exponents. Example (1) (a) Simplify (b) Simplify Example (2) Simplify Example (3) Simplify Note the following differences carefully: (-5x) 0 = 1, but -5x 0 = -5. Similarly: (-5)0 = 1, but -50 = -1. Example (4) Simplify (2a + b-1) -2 Exercises Q1 (5an-2) -1 Q2 Q3 (2a - b-2) -1 2.2 Fractional Exponents Fractional exponents can be used instead of using the radical sign (√). We use fractional exponents because often they are more convenient, and it can make algebraic operations easier to follow. Fractional Exponent Laws The n-th root of a number can be written using the power 1/n, as follows: Meaning: The n-th root of a when multiplied n times, gives us a. a 1/n × a 1/n × a 1/n × ... × a 1/n = a Definitions: The number under the radical is called the radicand (in the above case, the number a), and the number indicating the root being taken is called the order (or index) of the radical (in our case n). Example The 4-th root of 625 can be written as either: 6251/4 or equivalently, as Its value is 5, since 54 = 625. Raising the n-th root to the Power m If we need to raise the n-th root of a number to the power m (say), we can write this as: In English, this means "take the n-th root of the number, then raise the result to the power m". Example 1 Simplify Example 2 (a) Simplify (b) Simplify Example (3) Simplify Exercises Q1 Q2 2.3 Simplest Radical Form Before we can simplify radicals, we need to know some rules about them. These rules just follow on from what we learned in the first 2 sections in this chapter, Integral Exponents and Fractional Exponents. Expressing in simplest radical form just means simplifying a radical so that there are no more square roots, cube roots, 4th roots, etc left to find. It also means removing any radicals in the denominator of a fraction. Laws of Radicals n-th root of a Number to the Power n We met this idea in the last section, Fractional Exponents. Basically, finding the n-th root of a number is the opposite of raising the number to the power n, so they effectively cancel each other out. These 4 expressions have the same value: For the simple case, these all have the same value: √a 2 = (√a) 2 = √(a 2) = a The Product of the n-th root of a and the n-th root of b is the n-th root of ab We could write this using fractional exponents as well: The m-th Root of the n-th Root of the

Number a is the mn-th Root of a The equivalent expression using fractional exponents is as follows: The n-th Root of a Over the n-th Root of b is the n-th Root of a/b (b ≠ 0) If we write the same thing using fractional exponents, we have: (b ≠ 0) Example 1. Simplify the following: (a) Answer: We have used the first law above. (b) Answer: We have used . (c) Answer: We have used the law: (d) Answer: Nothing much to do here. We used: Example 2 In these examples, we are expressing the answers in simplest radical form, using the laws given above. (a) Answer: We need to examine 72 and find the highest square number that divides into 72. (Squares are the numbers 12 = 1, 22 = 4, 32 = 9, 42 = 16, ...) In this case, 36 is the highest square that divides into 72 evenly. We express 72 as 36 × 2 and proceed as follows. We have used the law: (b) Answer: We have used the law: √a 2 = a (c) Answer: (d) Answer: Exercises. Simplify: Q1 Q2 Q3 This one requires a special trick. To remove the radical in the denominator, we need to multiply top and bottom of the fraction by the denominator. 2.4 Addition and Subtraction of Radicals In algebra, we can combine terms that are similar eg. 2a + 3a = 5a 8x 2 + 2x − 3x 2 = 5x 2 + 2x Similarly for surds, we can combine those that are similar. They must have the same radicand (number under the radical) and the same index (the root that we are taking). Example 1 (a) Answer: We can do this because the radicand is 7 in each case and the index is 1/2 (that is, we are taking square root) in each case. (b) Answer: (c) Answer: Example 2 (a) Answer: In each part, we are taking square root, but the number under the square root is different. We need to simplify the radicals first and see if we can combine them. (b) Answer: Example 3 Simplify: Answer: Our aim here is to remove the radicals from the denominator of each fraction and then to combine the terms into one expression. We multiply top and bottom of each fraction with their denominators. This gives us a perfect square in the denominator in eachcase, and we can remove the radical. We then simplify and see that we have like terms (√6a). We then proceed to subtract the fractions by finding a common denominator (3a). Exercises Q1) Q2) Q3) 2.5 Multiplication and Division of Radicals When multiplying expressions containing radicals, we use the law , along with normal procedures of algebraic multiplication. Example 1 (a) Answer: (b) Answer: Example 2 (a) Answer: In this case, we needed to find the largest cube that divides into 24. (The cubes are the numbers 13 = 1, 23 = 8, 33 = 27, 43 = 64, ...) (b) Answer: Example 3 (a) Answer: (b) Answer: Division of Radicals (Rationalizing the Denominator) This process is also called "rationalising the denominator" since we remove all irrational numbers in the denominator of the fraction. This is important later when we come across Complex Numbers. Reminder: From earlier algebra, you will recall the difference of squares formula: (a + b)(a − b) = a 2 − b2 We will use this formula to rationalize denominators. Example Simplify: Answer The question requires us to divide 1 by (√3 − √2). We need to multiply top and bottom of the fraction by the conjugate of (√3 − √2). The conjugate is easily found by reversing the sign in the middle of the radical expression. In this case, our minus becomes plus. So the conjugate of (√3 − √2) is (√3 + √2). After we multiply top and bottom by the conjugate, we see that the denominator becomes free of radicals (in this case, the denominator has value 1). Historical Note In the days before calculators, it was important to be able to rationalize denominators. Using logarithm tables, it was very troublesome to find the value of expressions like our example above. Now that we use calculators, it is not so important to rationalize denominators. However, rationalizing denominators is still used for several of our algebraic techniques (see especially Complex Numbers), and is still worth learning. Exercises Q1) Q2) Q3) Q4) 2.6 Equations With Radicals It is important in this section to check your solutions in the original equation, as we often produce solutions which actually don't work when substituted back into the original equation. (In fact, it is always good to check solutions for equations - you learn so much more about why things are working the way they do.) Example: Solve Answer Squaring both sides gives: So our solutions are x = 1.5 or -1. CHECK: Substituting x = 1.5 in our original equation gives: Checks OK Substituting x = -1 in our original equation gives: DOES NOT check OK So we conclude the only solution is x = 1.5. Can you figure out where the 'wrong' solution is coming from? Exercises 1. Solve 2. Solve 3. Solve 4. In the study of spur gears in contact, the equation is used. Solve for .3.- Procedures and attitudes 1.- Students should be asked to recall the basic laws of exponents and to demonstrate their understanding of integral exponents. This could be done as a class activity. These could be listed as they are completed, so all students will have the opportunity to recall the basic facts. The students may be asked to consider the likely values of an expression of the type 251/2. Students could discuss possible answers in small groups and be instructed to provide a rationale for their group answer. If no group seems to approach the solution, the teacher could provide a series of clues for the students to utilize, beginning with the properties of exponents. E.g.: 251/2 = (?)1/2 = (5^2)1/2 = (5)2(1/2) = 5^1 = 5 In the above example, the students should develop, or be shown, the relationship of an exponent of 1/2 and the square root. The notation for a rational exponent can be introduced, (x1/m = m x 1), and several examples done by the teacher, class, or individual students. The definition can be extended to (Xn/m = m Xn) and more examples considered. 2.- Some statements could be given to the students to work on in pairs or in small groups. These should give the students some practice in working with rational exponents. E.g: a) x2/3 · x 5/3 b) (x1/2) 2/3 Once the students have had practice in working with the laws of exponents, this can be extended to simplification, or evaluation exercises as well. E.g.: x1/2 · x 1/3 81/2 · 81/6 3.- Students will be expected to be able to demonstrate that they understand the definition of a rational exponent by writing a given radical expression in radical form. Students should be instructed to review their definition of a rational exponent and to apply that definition to a set of exercises in which they would practise this concept. They may work individually, in pairs, small groups, or as an entire class, in practising this concept. 4.- A review of the simplifying process of square roots as studied in Mathematics 20 may be a useful starting point. When the students have recalled the principles of simplifying square root radicals, they can be introduced to cube root simplification. It may be necessary to have the students compare the square root and the cube root of various numbers, to help them discover that the cube root of a negative number does in fact exist. This can be tied to the definition of a rational exponent and aid understanding of the role of an even or odd denominator in rational exponents. There are several different methods by which this concept can be explained; check your reference texts for different explanations. Some students may prefer to look at some of these options. Note that one common error that students tend to make with is (a + b) = a + b Try some numerical examples to show this is not the case. Ask students if there are any cases where (a + b) = a + b? 5.- Students are expected to become familiar with converting from exponential expressions to radical form and vice versa. In objective D.3, the former was introduced and practice given. The teacher may find it useful to combine objectives D.3 and D.5 in one lesson. In any case, some practice should be given to the students to work on converting from radical to exponential form. The students could practise individually, in pairs, or in small groups. 6.- Square root operations were introduced in third year. Students may be asked to recall the basic procedures involved in these operations. Students can be given a few exercises to do to review these third year operations. The introduction of cube root operations can be done as an entire class or by getting individual or group input when developing these procedures. Students could be given a set of exercises to use in developing their skills in dealing with square and cube root operations. Students could work in pairs or in small groups in doing these exercises. 7.- Students can be introduced to the process of rationalizing by practicing selected exercises, and being asked to examine the results obtained in these situations. E.g.: 5 * 5 = 3 7 * 3 7 * 3 7 = (3 2 - 5)(3 2 - 5) = They can be asked to determine an expression that could be multiplied by a given expression so that the result is a rational number. E.g.: (4 5 - 3 7)(?) a rational result. Once the basic process of converting an irrational denominator to a rational denominator is observed by the students, they could be introduced to exercises which involve rationalizing the denominator. They could work on exercises individually, in pairs, in small groups, or as a class. Students should be expected to summarize their results, and describe the processes utilized for denominators with square roots, and for denominators with cube roots. The teacher may wish to have students theorize about a process that could be utilized for denominators with other indices. 8.- Students have already solved quadratic equations by factoring, and by taking the square root of both sides of the equation, in third year. As well, they have already reviewed these types previously in fourth year. Students should be given some practice in factoring perfect trinomial squares, and in determining the value of c in ax^2+ bx+c, which would make ax^2+bx+c a perfect square. They could work cooperatively on these skills. Students should be able to summarize their findings, draw conclusions about the completion of a perfect square, and demonstrate their understanding by completing other examples. Once students are able to complete the square, this process can be utilized in solving quadratic equations. Students should observe some examples and work cooperatively to solve (and check) several examples of their own. The quadratic formula can then be developed as an exercise in solving quadratic equations by completing the square for the general case ax^2 +bx+c. Students can attempt the general case in small groups, with the teacher providing assistance and hints as necessary. Students should have time in which to practise using all of these methods in solving (and checking) quadratic equations. 9.- Students were introduced to radical equations involving one radical. A brief review of one or two exercises may be useful in introducing this topic. Most textbooks use an algorithmic approach to solving radical equations involving two radicals, and this approach may be utilized in the classroom. Students can be given a few instructions, and then be asked to complete a few exercises, working cooperatively in small groups or individually, to solve these exercises. After a short time, answers or possible solutions can be shared with the entire class. It should be pointed out that checking the solutions is important and the checking procedure should be carried out when solutions are presented to the class. Students should be asked to observe the solutions to determine if there are any general procedures that might be followed which make the solution to these equations simpler. Any such observations can be discussed with the entire class. 10.- Once suitable word problems have been identified, students should work cooperatively to determine the necessary information, to pose the question to be answered and to write the equation, which, when solved, will provide the solution to the problem. Problems may be presented one or two at a time. Student answers may be shared with the entire class after students have had a few moments to work on the problems. A discussion of the main components of the problem, and the setup of the equation may help all students increase their skills in problem solving. A few problems may be assigned to the class to complete in small groups. Some statement should be elicited from the student as to whether the problem outlined in each case is a real-world situation, and if so, who might be responsible for its solution in its real-world context.

Topic: Sequence

Arithmetic Sequence

Arithmetic Mean

The two simplest sequences to work with are arithmetic and geometric sequences.

An arithmetic sequence goes from one term to the next by always adding (or subtracting) the same value. For instance, 2, 5, 8, 11, 14,... is arithmetic, because each step adds three; and 7, 3, –1, –5,... is arithmetic, because each step subtracts 4.

The number added (or subtracted) at each stage of an arithmetic sequence is called the "common difference" *d*, because if you subtract (that is, if you find the difference of) successive terms, you'll always get this common value.

A geometric sequence goes from one term to the next by always multiplying (or dividing) by the same value. So 1, 2, 4, 8, 16,... is geometric, because each step multiplies by two; and 81, 27, 9, 3, 1, \frac{1}{3}31,... is geometric, because each step divides by 3.

The number multiplied (or divided) at each stage of a geometric sequence is called the "common ratio" *r*, because if you divide (that is, if you find the ratio of) successive terms, you'll always get this common value.

- *Find the common difference and the next term of the following sequence:*

3, 11, 19, 27, 35, ...

To find the common difference, I have to subtract a successive pair of terms. It doesn't matter which pair I pick, as long as they're right next to each other. To be thorough, I'll do all the subtractions:

11 – 3 = 8

19 – 11 = 8

27 – 19 = 8

35 – 27 = 8

The difference is always 8, so the common difference is $d = 8$.

They gave me five terms, so the sixth term of the sequence is going to be the very next term. I find the next term by adding the common difference to the fifth term:

35 + 8 = 43

Then my answer is:

common difference: $d = 8$

sixth term: 43

e value. So 1, 2, 4, 8, 16,... is geometric, because each step multiplies by two; and 81, 27, 9, 3, 1, \frac{1}{3}31,... is geometric, because each step divides by 3.

The number multiplied (or divided) at each stage of a geometric sequence is called the "common ratio" r, because if you divide (that is, if you find the ratio of) successive terms, you'll always get this common value.

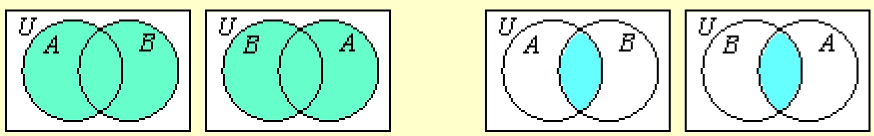
Topic: Operation on Sets

Properties of Union and Intersection of Sets

The following set properties are given here in preparation for the properties for addition and multiplication in arithmetic. Note the close similarity between these properties and their corresponding properties for addition and multiplication.

Commutative Properties: The *Commutative Property for Union* and the *Commutative Property for Intersection* say that the order of the sets in which we do the operation does not change the result.

General Properties: $A \cup B = B \cup A$ and $A \cap B = B \cap A$.



Example: Let $A = \{x : x$ is a whole number between 4 and 8$\}$ and $B = \{x : x$ is an even natural number less than 10$\}$.

Then $A \cup B = \{5, 6, 7\} \cup \{2, 4, 6, 8\} = \{2, 4, 5, 6, 7, 8\} = \{2, 4, 6, 8\} \cup \{5, 6, 7\} = B \cup A$

and $A \cap B = \{5, 6, 7\} \cap \{2, 4, 6, 8\} = \{6\} = \{2, 4, 6, 8\} \cap \{5, 6, 7\} = B \cap A$.

Associative Properties: The *Associative Property for Union* and the *Associative Property for Intersection* says that how the sets are grouped does not change the result.

General Property: $(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cap C = A \cap (B \cap C)$

Example: Let $A = \{a, n, t\}$, $B = \{t, a, p\}$, and $C = \{s, a, p\}$.

Then $(A \cup B) \cup C = \{p, a, n, t\} \cup \{s, a, p\} = \{p, a, n, t, s\} = \{a, n, t\} \cup \{t, a, p, s\} = A \cup (B \cup C)$

and $(A \cap B) \cap C$ = {a, t} ∩ {s, a, p} = {a} = {a, n, t} ∩ {a, p} $= A \cap (B \cap C)$.

Identity Property for Union: The *Identity Property for Union* says that the union of a set and the empty set is the set, i.e., union of a set with the empty set includes all the members of the set.

General Property: $A \cup \emptyset = \emptyset \cup A = A$

Example: Let A = {3, 7, 11} and B = {x : x is a natural number less than 0}.
Then $A \cup B$ = {3, 7, 11} ∪ { } = {3, 7, 11}.

The empty set is the identity element for the union of sets. What would be the identity element for the addition of whole numbers? What would be the identity element for multiplication of whole numbers?

Intersection Property of the Empty Set: The *Intersection Property of the Empty Set* says that any set intersected with the empty set gives the empty set.

General Property: $A \cap \emptyset = \emptyset \cap A = \emptyset$.

Example: Let A = {3, 7, 11} and B = {x : x is a natural number less than 0}.
Then $A \cap B$ = {3, 7, 11} ∩ { } = { }.

What number has a similar property when multiplying whole numbers? What is the corresponding property for multiplication of whole numbers?

Distributive Properties: The *Distributive Property of Union over Intersection* and the *Distributive Property of Intersection over Union* show two ways of finding results for certain problems mixing the set operations of union and intersection.

General Property: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Example: Let A = {a, n, t}, B = {t, a, p}, and C = {s, a, p}. Then

$A \cup (B \cap C)$ = {a, n, t} ∪ {a, p} = {p, a, n, t} = {p, a, n, t} ∩ {p, a, n, t, s} = $(A \cup B) \cap (A \cup C)$

$A \cap (B \cup C)$ = {a, n, t} ∩ {t, a, p, s} = {a, t} = {a, t} ∪ {a} = $(A \cap B) \cup (A \cap C)$

Section 1.4 Operations with sets – Union, Intersection and Complement A universal set for a particular problem is a set which contains all the elements of all the sets in the problem. A universal set is often denoted by a capital U, but sometimes the Greek letter ξ (xee) is used. In this section we will create subsets of a given universal set and use set operations to create new subsets of the universal set. There are three set operations we will learn in this section. • Complement: The complement of a set A is symbolized by A' and it is the set of all elements in the universal set that are not in A. • Intersection: The intersection of sets A and B is symbolized by $A \cap B$ and is the set containing all of the elements that are common to both set A and set B. • Union: The union of set A and B is symbolized $A \cup B$ and is the set containing all the elements that are elements of set A or of set B or that are in both Sets A and B. Here is a quick example to illustrate the 3 definitions. Example: Let U be a universal set and A and B be subsets of U defined as follows. U = {1,2,3,4,5} A = {1,2,3} B = {2,3,4} Find A' A' is all of the elements in the Universal set that are not in set A. Answer: A' = {4,5} Find $\boldsymbol{A} \cap \boldsymbol{B}$ (This is asking me to find all of the elements that A and B have in common.) Answer: $A \cap B$ = {2,3} Find $\boldsymbol{A} \cup \boldsymbol{B}$ (This is asking me to list all of the elements in A followed by all of the elements in B, then delete any elements that are written twice.) $A \cup B$ = {1,2,3,2,3,4} Answer: $A \cup B$ = {1,2,3,4} Example: Let U be a universal set and A and B be subsets of U defined as follows. U = {a,b,c,d,e,f} A = {a,b,c} B = {c,d,e} Find $\boldsymbol{A}' \cap \boldsymbol{B}$ First I need to find A', which is all of the elements in U that aren't in set A. A' = {d,e,f} Now I can intersect the two sets. $A' \cap B$ = {d,e,f} ∩ {c,d,e} (now find what the two sets have in common) Answer: {d,e} Find $\boldsymbol{A} \cup \boldsymbol{B}'$ First I need to find B' B' = {a,b,f} A ∪ B′ = {a,b,c} ∪ {a,b,f} (put all 6 elements in a big set then delete the duplicates) = {a,b,c,a,b,f} Answer: {a,b,c,f} #1-10: Find the following sets. U = {a,b,c,d,e} A = {c,d,e} B = {a,c,d} 1) A' 2) B' 3) $A \cup B$ 4) $A' \cup B'$ 5) $A \cap B$ 6) $A' \cap B'$ 7) $A' \cap B$ 8) $A \cap B'$ 9) $A' \cup B$ 10) $A \cup B'$ #11-20: Find the following sets. U = {1,2,3,4,5} A = {1,2,3} B = {5} 11) A' 12) B' 13) $A \cup B$ 14) $A' \cup B'$ 15) $A \cap B$ 16) $A' \cap B'$ 17) $A' \cap B$ 18) $A \cap B'$ 19) $A' \cup B$ 20) $A \cup B'$ Example: Let U be a universal set and A, B and C be subsets of U defined as follows. U = {a,b,c,d,e,f} A = {a,b,c} B = {c,d,e} C = {d,e,f} Find $\boldsymbol{A} \cup \boldsymbol{B} \cup \boldsymbol{C}$ I need to work from left to right. First I will find $A \cup B A \cup B$ = {a,b,c} ∪ {c,d,e} = {a,b,c,c,d,e} = {a,b,c,d,e} Now I can do the union C part. I can rewrite my problem as: {a,b,c,d,e} ∪ C = {a,b,c,d,e} ∪ {d,e,f} = {a,b,c,d,e,d,e,f} Answer: {a,b,c,d,e,f} Find $(\boldsymbol{B} \cup \boldsymbol{C})'$ I have to work on the inside of the parenthesis first. So I will first find: $B \cup C B \cup C$ = {c,d,e} ∪ {d,e,f} $B \cup C$ = {c,d,e,d,e,f} $B \cup C$ ={c,d,e,f} Now I can do the complement. I can replace the inside of the parenthesis with {c,d,e,f} and proceed to find its complement. (B ∪ C)′ = (c,d,e,f)' (my answer will be all the elements of set U that are not in this set.) Answer: {a,b} Find $\boldsymbol{A} \cup (\boldsymbol{B} \cup \boldsymbol{C})'$ First I need to simplify the parenthesis (B ∪ C)′ I just figured out that (B ∪ C)′ = {a,b}, so I will use the work I have already done A ∪ (B ∪ C)′ = A ∪ {a, b} = {a,b,c} ∪ {a,b} = {a,b,c,a,b} Answer: {a,b,c} Find $A' \cap (B \cap C')$ I need to simplify the inside of the parenthesis first. $(B \cap C')$ = {c,d,e} ∩ {a,b,c} = {c} $A' \cap (B \cap C')$ = A' ∩ $\{c\}$ = {d,e,f} ∩ {c} Answer: ∅ (empty set) #21-32: Find the following sets. U = {1,2,3,4,5,6} A = {1,2,3} B = {2,3,4} C = {1,5} 21) $A \cap C$ 22) $B \cap C$ 23) $A \cup C$ 24) $B \cup C$ 25) $A \cap B \cup C$ 26) $A \cup B \cap C$ 27) $B \cup C \cap A$ 28) $B \cap A \cup C$ 29) $A' \cap B$ 30) $A \cap B'$ 31) $A' \cup B \cap C'$ 32) $B' \cap A \cup C'$ #33-44: Find the following sets. U = {a,b,c,d} A = {a,b,c} B = {b,c,d} C = {a,d} 33) $A \cap C'$ 34) $B' \cap C$ 35) $A' \cup C'$ 36) $B' \cup C'$ 37) $A' \cap B \cup C'$ 38) $A' \cup B' \cap C$ 39) $B' \cup C' \cap A$ 40) $B' \cap A' \cup C$ 41) $A' \cap B'$ 42) $A \cap B'$ 43) $A' \cup B' \cap C'$ 44) $B \cap A' \cup C'$ #45 – 56: Find the following sets. U = {1,2,3,4,5,6} S = {2,4,6} T = {1,2,4} V = {4,5,6} 45) $S \cup (T \cap V)$ 46) $(S \cup T) \cap V$ 47) $(S \cup T)'$ 48) $(V \cup S)'$ 49) $S \cap (V \cap T')$ 50) $(S' \cap V') \cup T$ 51) $(S' \cup V') \cap T$ 52) $S' \cup T \cap V'$ 53) $T \cup V' \cup S'$ 54) $T \cup V' \cap S'$ 55) $(V \cap T)' \cup S$ 56) $V \cup (S \cap T)'$ Answers: 1) {a,b} 3) {a,c,d,e} 5) {c,d} 7) {a} 9) {a,b,c,d} 11) {4,5} 13) {1,2,3,5} 15) ∅ 17)

{5} 19) {4,5} 21) {1} 23) {1,2,3,5} 25) {1,2,3,5} 27) {1,2,3} 29) {4} 31) {2,3,4,6} 33) {b,c} 35) {b,c,d} 37) {b,c,d} 39) {a,b,c} 41) ∅ 43) ∅ 45) {2,4,6} 47) {3,5} 49) {6} 51) {1,2} 53) {1,2,3,4,5} 55) {1,2,3,4,5,6}

Topic: Function

Types of Functions

What does the word function stand for? By the word function, we understand the responsibility or role one has to play. What is the function of the leaves of plants – to prepare food for the plant and store them? What is the function of the roots of plants? They absorb water and other nutrients from the ground and supply it to the plants and help them stand erect. Can we say that everyone has different types of functions? Let's learn about some types of function in mathematics!

Suggested Videos

Cartesian Product

Introduction to Relations

Functions

Functions

We can define a function as a special relation which maps each element of set A with one and only one element of set B. Both the sets A and B must be non-empty. A function defines a particular output for a particular input. Hence, f: A ⟶ B is a function such that for a ∈ A there is a unique element b ∈ B such that (a, b) ∈ f

Browse more Topics under Relations And Functions

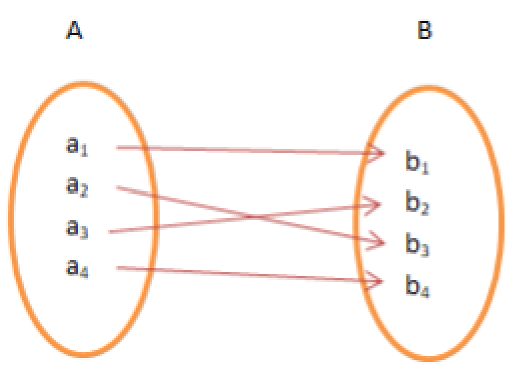
- Relations
- Functions
- Types of Relations
- Representation of Functions
- Composition of Functions and Invertible Function
- Algebra of Real Functions
- Cartesian Product of Sets
- Binary Operations

Types of Functions

We have already learned about some types of functions like Identity, Polynomial, Rational, Modulus, Signum, Greatest Integer functions. In this section, we will learn about other types of function.

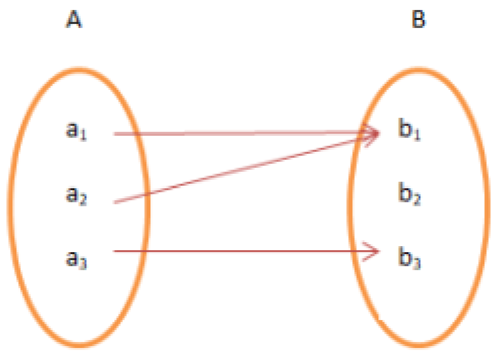
One to One Function

A function f: A ⟶ B is One to One if for each element of A there is a distinct element of B. It is also known as Injective. Consider if $a_1 \in A$ and $a_2 \in B$, f is defined as f: A ⟶ B such that f (a_1) = f (a_2)



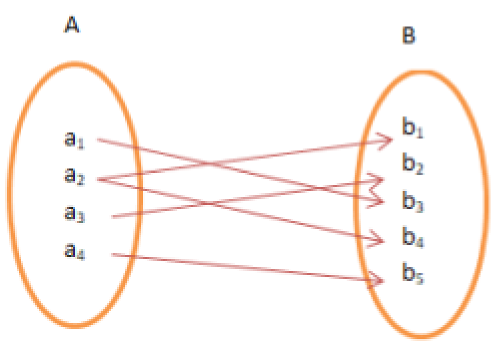
Many to One Function

It is a function which maps two or more elements of A to the same element of set B. Two or more elements of A have the same image in B.



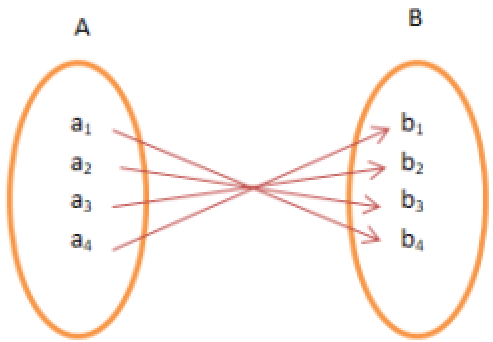
Onto Function

If there exists a function for which every element of set B there is (are) pre-image(s) in set A, it is Onto Function. Onto is also referred as Surjective Function.



One – One and Onto Function

A function, f is One – One and Onto or Bijective if the function f is both One to One and Onto function. In other words, the function f associates each element of A with a distinct element of B and every element of B has a pre-image in A.



Browse more topics under Relations and Functions

Relations and Functions

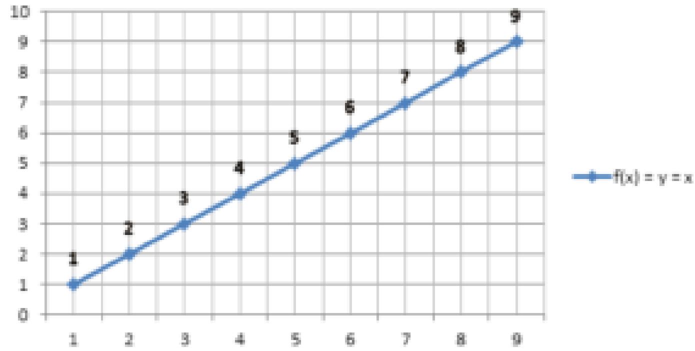
- Binary Operations
- Cartesian Product of Sets
- Algebra of Real Functions
- Composition of Functions and Invertible Function
- Representation of Functions
- Types of Relations
- Functions
- Relations

Other Types of Functions

A function is uniquely represented by its graph which is nothing but a set of all pairs of x and f(x) as coordinates. Let us get ready to know more about the types of functions and their graphs.

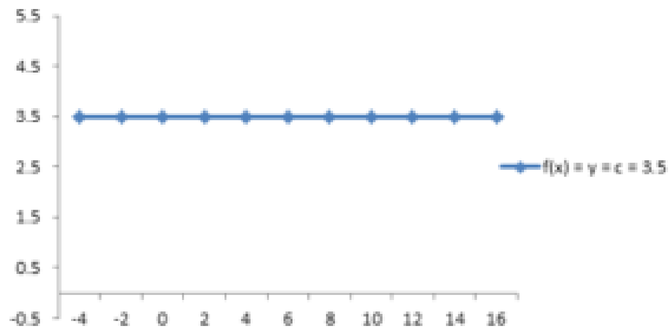
Identity Function

Let **R** be the set of real numbers. If the function f: **R**→**R** is defined as f(x) = y = x, for x ∈ R, then the function is known as Identity function. The domain and the range being **R.** The graph is always a straight line and passes through the origin.



Constant Function

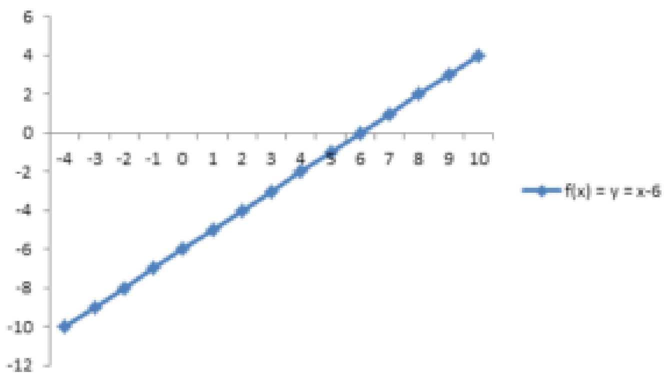
If the function f: **R**→**R** is defined as f(x) = y = c, for x ∈ R and c is a constant in **R**, then such function is known as Constant function. The domain of the function f is **R** and its range is a constant, c**.** Plotting a graph, we find a straight line parallel to the x-axis.



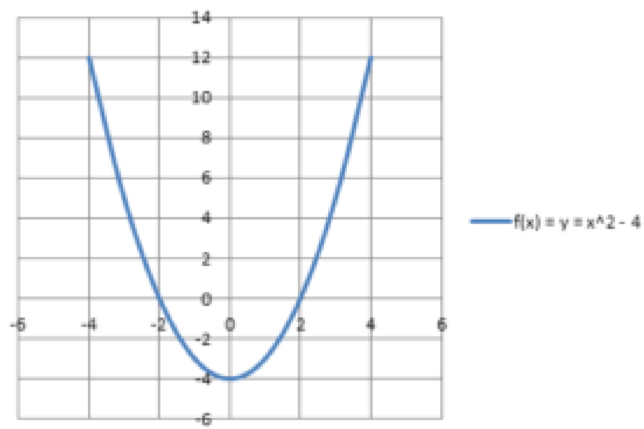
Polynomial Function

A polynomial function is defined by $y = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$, where n is a non-negative integer and $a_0, a_1, a_2, \ldots, {}_n \in$ **R**. The highest power in the expression is the degree of the polynomial function. Polynomial functions are further classified based on their degrees:

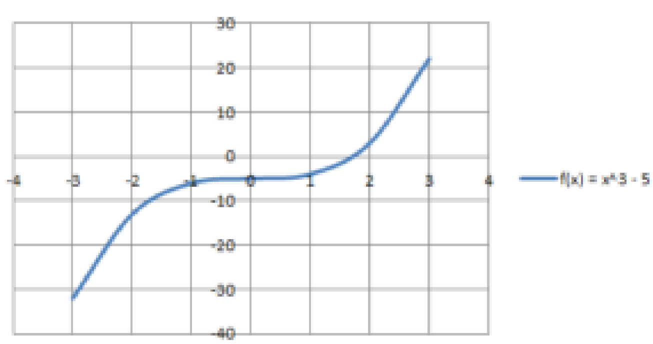
- Constant Function: If the degree is zero, the polynomial function is a constant function (explained above).
- Linear Function: The polynomial function with degree one. Such as y = x + 1 or y = x or y = 2x – 5 etc. Taking into consideration, y = x – 6. The domain and the range are **R.** The graph is always a straight line.



Quadratic Function: If the degree of the polynomial function is two, then it is a quadratic function. It is expressed as $f(x) = ax^2 + bx + c$, where $a \neq 0$ and a, b, c are constant & x is a variable. The domain and the range are **R.** The graphical representation of a quadratic function say, $f(x) = x^2 – 4$ is

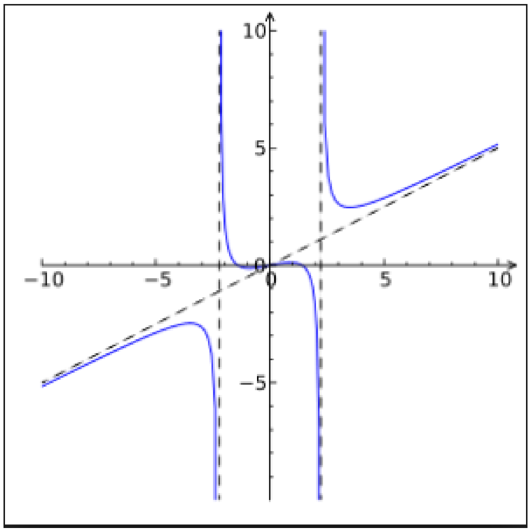


- Cubic Function: A cubic polynomial function is a polynomial of degree three and can be denoted by $f(x) = ax^3 + bx^2 + cx +d$, where $a \neq 0$ and a, b, c, and d are constant & x is a variable. Graph for $f(x) = y = x^3 – 5$. The domain and the range are **R.**



Rational Function

A rational function is any function which can be represented by a rational fraction say, f(x)/g(x) in which numerator, f(x) and denominator, g(x) are polynomial functions of x, where $g(x) \neq 0$. Let a function f: **R** → **R** is defined say, $f(x) = 1/(x + 2.5)$. The domain and the range are **R**. The Graphical representation shows asymptotes, the curves which seem to touch the axes-lines.

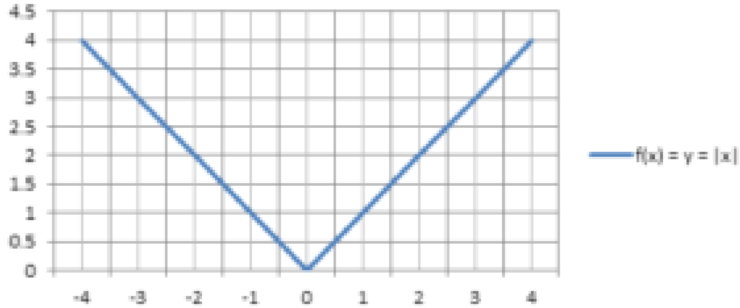


Modulus Function

The absolute value of any number, c is represented in the form of |c|. If any function f: **R**→ **R** is defined by f(x) = |x|, it is known as Modulus Function. For each non-negative value of x, f(x) = x and for each negative value of x, f(x) = -x, i.e.,

$f(x) = \{x, \text{ if } x \geq 0; -x, \text{ if } x < 0.$

Its graph is given as, where the domain and the range are **R.**

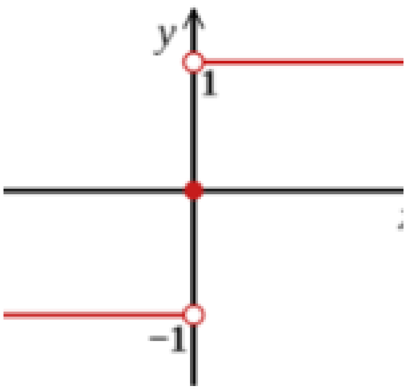


Signum Function

A function f: **R**→ **R** defined by

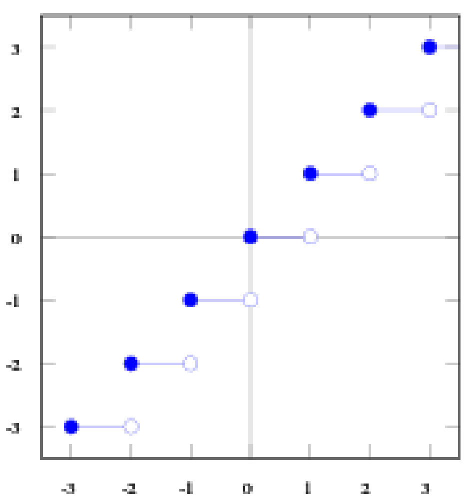
f(x) = { 1, if x > 0; 0, if x = 0; -1, if x < 0

Signum or the sign function extracts the sign of the real number and is also known as step function.



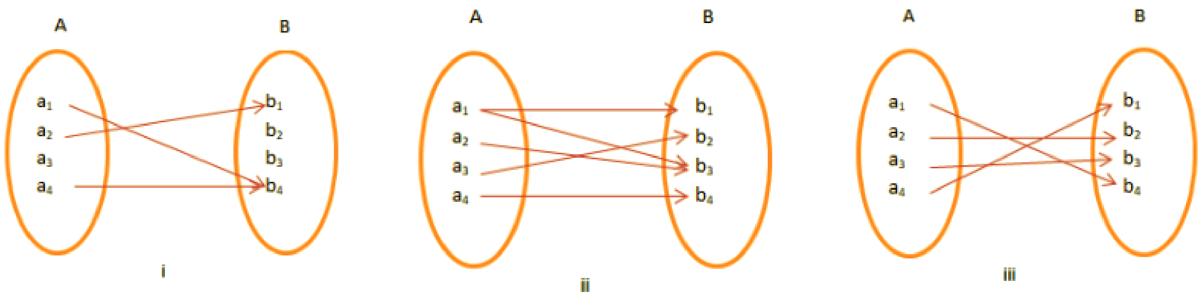
Greatest Integer Function

If a function f: **R**→ **R** is defined by f(x) = [x], x ∈ X. It round-off to the real number to the integer less than the number. Suppose, the given interval is in the form of (k, k+1), the value of greatest integer function is k which is an integer. For example: [-21] = 21, [5.12] = 5. The graphical representation is



Solved Example for You

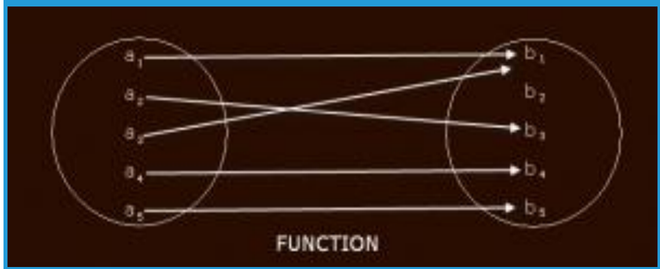
Question: Which of the following is a function?

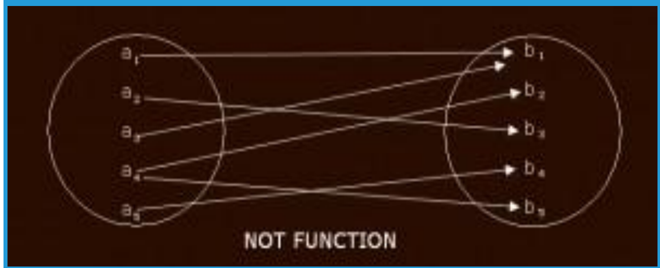


Solution: Figure (iii) is an example of a function. Since the given function maps every element of A with that of B. In figure (ii), the given function maps one element of A with two elements of B (one to many). Figure (i) is a violation of the definition of the function. The given function does not map every element of A.

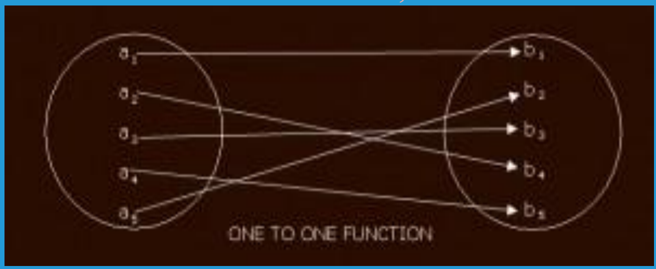
What is function? Function (mathematics) is defined as if each element of set A is connected with the elements of set B, it is not compulsory that all elements of set B are connected; we call this relation as function.

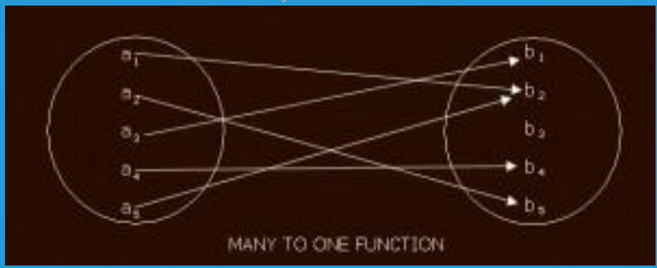
f: A ⟶ B (f is a function from A to B)



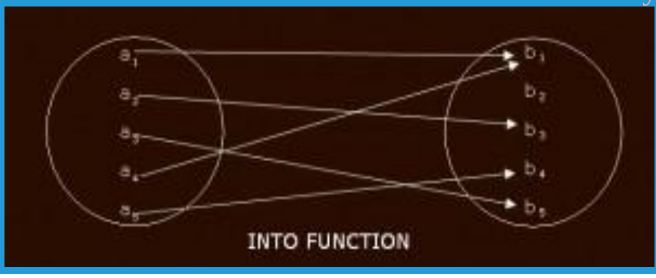


Types of function:

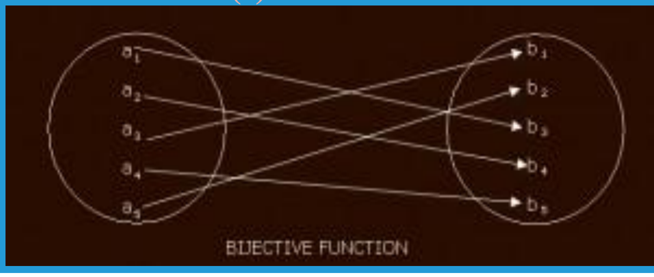
1. **One-one Function or Injective Function :** If each elements of set A is connected with different elements of set B, then we call this function as One-one function.


2. **Many-one Function :** If any two or more elements of set A are connected with a single element of set B, then we call this function as Many one function.


3. **Onto function or Surjectivefunction :** Function f from set A to set B is onto function if each element of set B is connected with set of A elements.


4. **Into Function :** Function f from set A to set B is Into function if at least set B has a element which is not connected with any of the element of set A.


5. **One-one Onto Function or Bijective function :** Function f from set A to set B is One one Onto function if(a) f is One one function(b) f is Onto function.



Topic: Cartesian Plane and Linear Graphs

Linear equations in the coordinate plane

A linear equation is an equation with two variables whose graph is a line. The graph of the linear equation is a set of points in the coordinate plane that all are solutions to the equation. If all variables represent real numbers one can graph the equation by plotting enough points to recognize a pattern and then connect the points to include all points.

If you want to graph a linear equation you have to have at least two points, but it's usually a good idea to use more than two points. When choosing your points try to include both positive and negative values as well as zero.

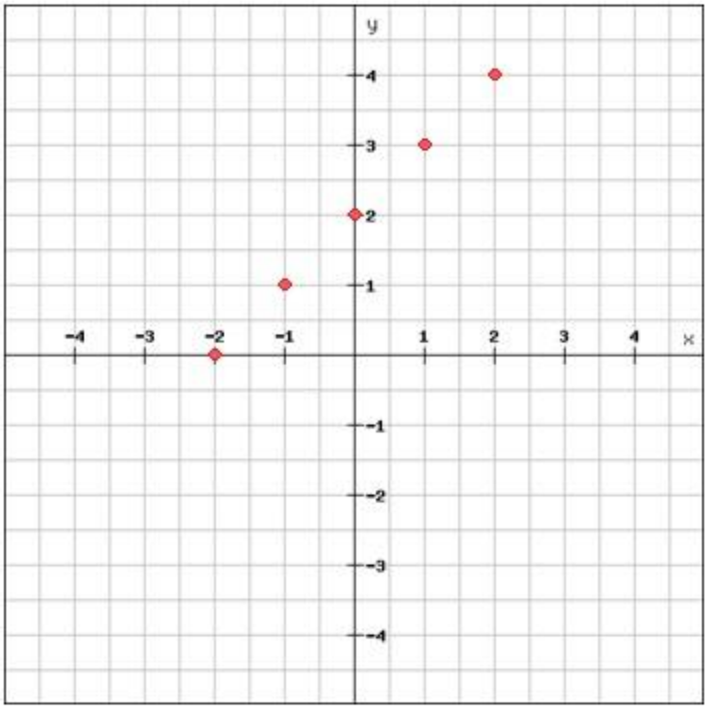
Example

Graft the function y = x + 2

Begin by choosing a couple of values for x e.g. -2, -1, 0, 1 and 2 and calculate the corresponding y values.

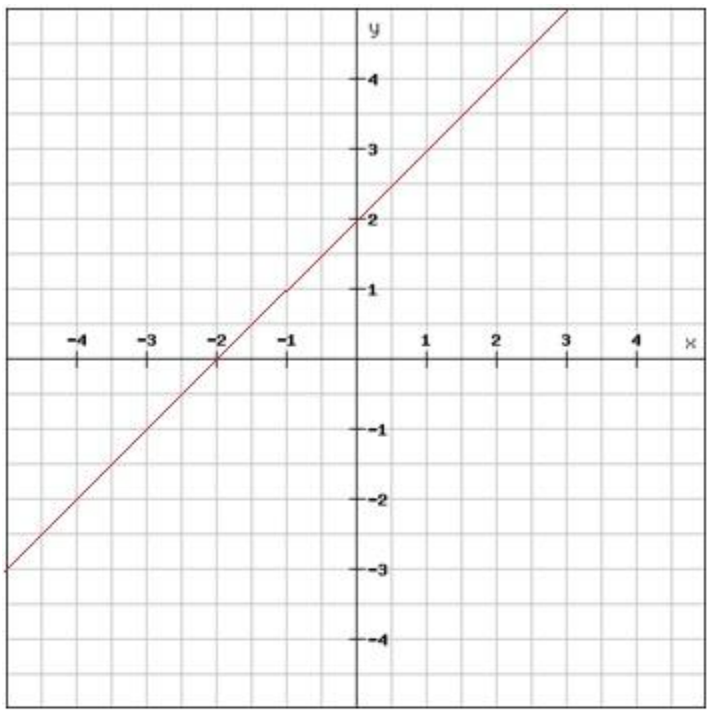
X	Y = x + 2	Ordered pair
-2	-2 + 2 = 0	(-2, 0)
-1	-1 + 2 = 1	(-1, 1)
0	0 + 2 = 2	(0, 2)
1	1 + 2 = 3	(1, 3)
2	2 + 2 = 4	(2, 4)

Now you can just plot the five ordered pairs in the coordinate plane

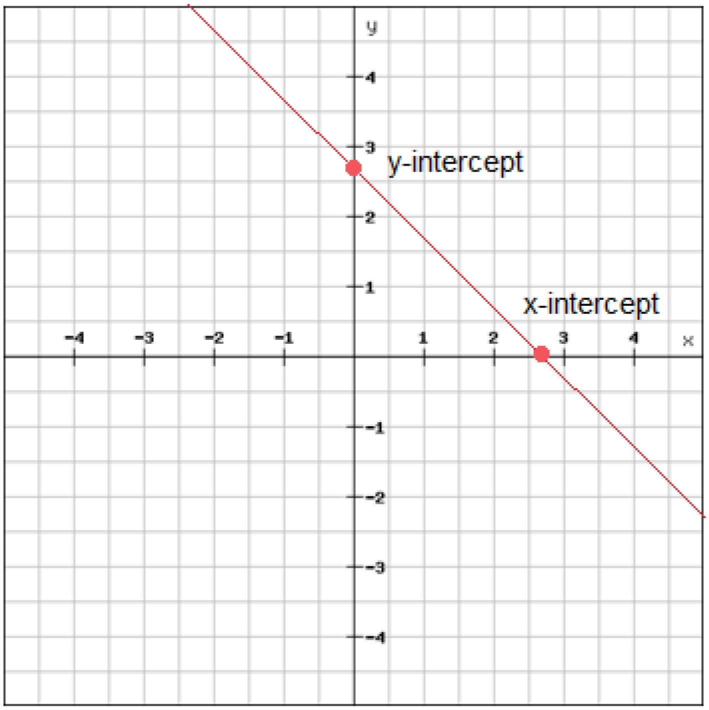


At the moment this is an example of a discrete function. A discrete function consists of isolated points.

By drawing a line through all points and while extending the line in both directions we get the opposite of a discrete function, a continuous function, which has an unbroken graph.



If you only want to use two points to determine your line you can use the two points where the graph crosses the axes. The point in which the graph crosses the x-axis is called the x-intercept and the point in which the graph crosses the y-axis is called the y-intercept. The x-intercept is found by finding the value of x when y = 0, (x, 0), and the y-intercept is found by finding the value of y when x = 0, (0, y).



The standard form of a linear equation is

$$Ax+By=C, A,B\neq 0$$

Before you can graph a linear equation in its standard form you first have to solve the equation for y.

$$2y-4x=8$$

$$2y-4x+4x=8+4x$$

$$2y=4x+8$$

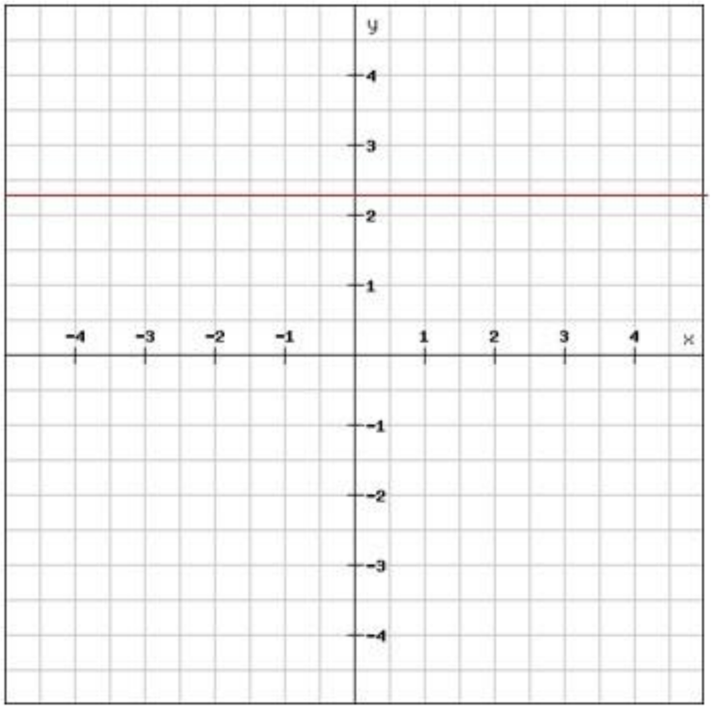
$$\frac{2y}{2}=\frac{4x}{2}+\frac{8}{2}$$

$$y=2x+4$$

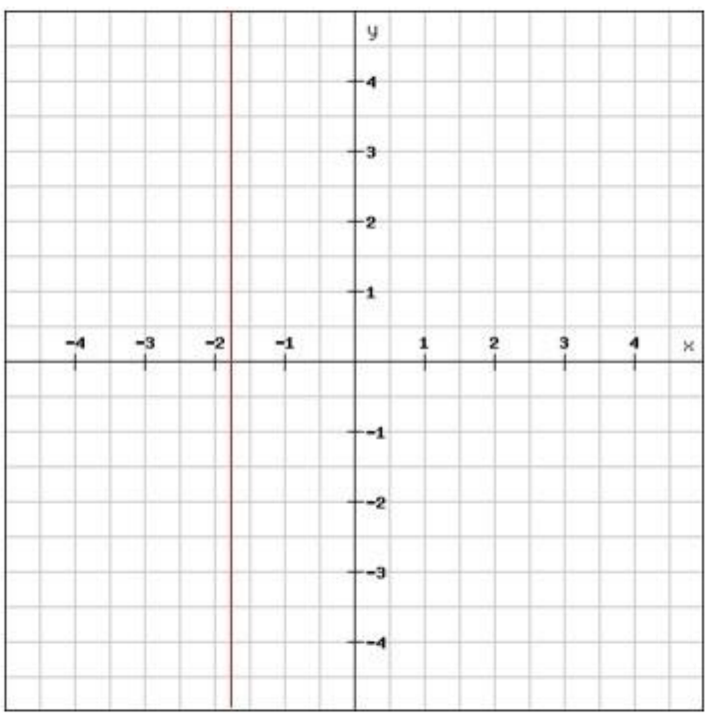
From here you can graph the equation as we did in the example above.

The graph of y = a is a horizontal line where the line passes through the point (0, a)

The coordinate plane

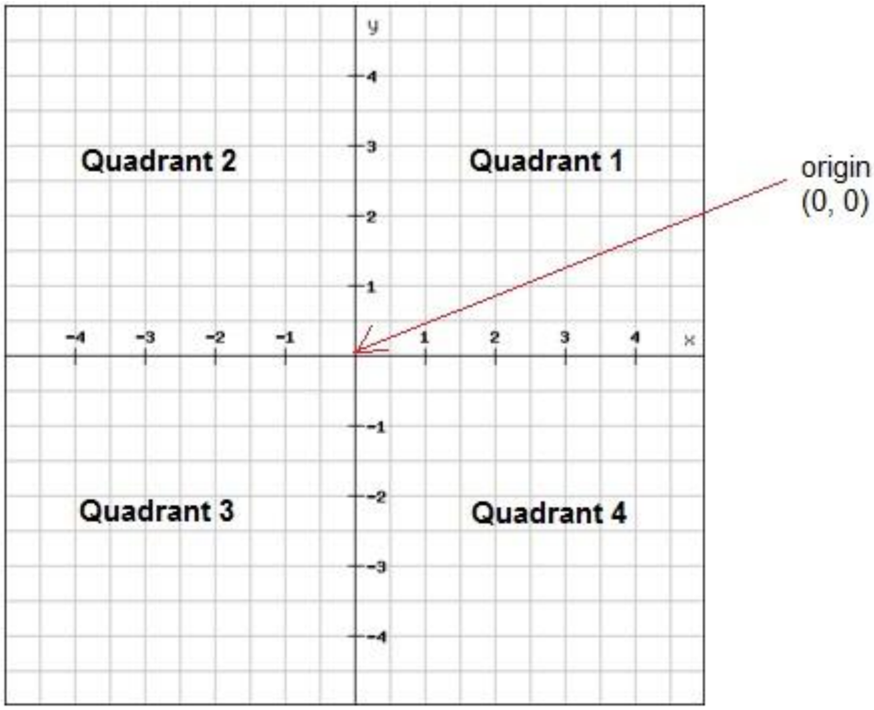


Whereas the graph of x = a is a vertical line that passes through the point (a, 0)



The coordinate plane

As you remember from pre-algebra a coordinate plane is a two-dimensional number line where the vertical line is called the y-axis and the horizontal is called the x-axis. These lines are perpendicular and intersect at their zero points. This point is called the origin. The axes divide the plane into four quadrants.



A point in a coordinate plane is named by its ordered pair of the form of (x, y). The first number corresponds to the x-coordinates and the second to the y-coordinate.

The completeness property for points in the plane tells us two things

1. Exactly one point in the plane is named given the numbers of the ordered pair and
2. Exactly one ordered pair of numbers names a given point in the plane.

To graph a point one draws a dot at the coordinates that corresponds to the ordered pair. It's always a good idea to start at the origin. The x-coordinate tells you have many steps you have to take to the right (positive) or left (negative). And the y-coordinate tells you have many steps to move up (positive) or down (negative).

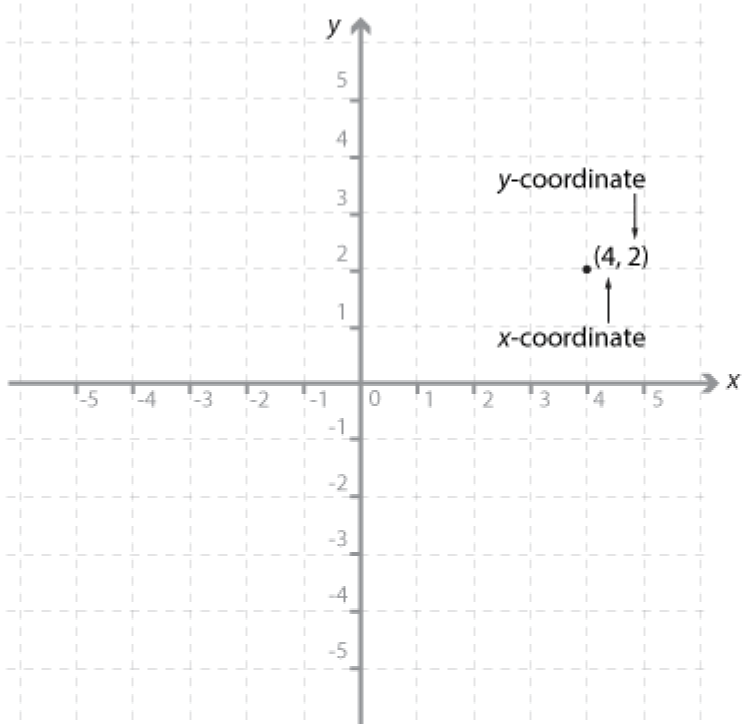
To find out the coordinates of a point in the coordinate system you do the opposite. Begin at the point and follow a vertical line either up or down to the x-axis. There is your x-coordinate. And then do the same but following a horizontal line to find the y-coordinate.

A relation is a set of ordered pairs. The first coordinate (usually the x-coordinate) is called the domain and the second (usually the y-coordinate) is called the range. If you remember from earlier chapters the domain contains values that correspond to the independent variable whereas the range contains values corresponding to the dependent variable.

Plotting linear relationships and examples of linear relations

- Introduction
- Teacher resources
- Student resources

Students should first plot points. In year 8, they plot points from tables of values of both functions. It is possible to introduce the concepts of gradient and y-intercept at this level.



Detailed description

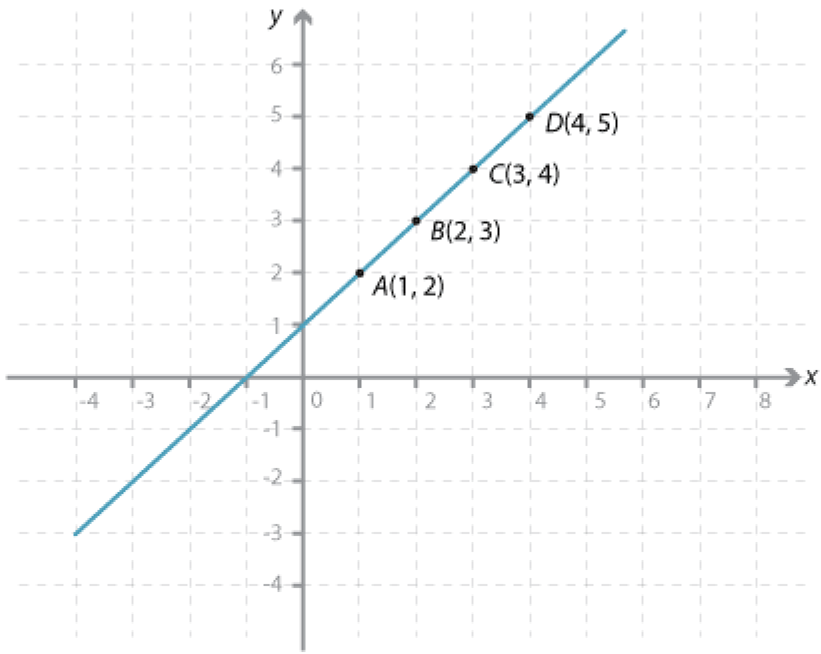
The number plane (Cartesian plane) is divided into four quadrants by two perpendicular axes called the x-axis (horizontal line) and the y-axis (vertical line). These axes intersect at a point called the origin. The position of any point in the plane can be represented by an ordered pair of numbers (x, y). These ordered pairs are called the coordinates of the point.

The point with coordinates (4, 2) has been plotted on the Cartesian plane shown. The coordinates of the origin are (0, 0).

Plotting points

We can plot sets of ordered pairs and look at the patterns that emerge.

Example 1



Plotting Linear Graphs

If the rule for a relation between two variables is given, then the graph of the relation can be drawn by constructing a table of values.

To plot a **straight line graph** we need to find the coordinates of *at least two points* that fit the rule.

Example 6

Plot the graph of $y = 3x + 2$.

Solution:

Construct a table and choose simple x values.

x	-2	-1	0	1	2
y					

In order to find the y values for the table, substitute each x value into the rule $y = 3x + 2$.

When $x = -2,\ y = 3(-2) + 2$
$= -6 + 2$
$= -4$

When $x = -1,\ y = 3(-1) + 2$
$= -3 + 2$
$= -1$

When $x = 0,\ y = 3 \times 0 + 2$
$= 0 + 2$
$= 2$

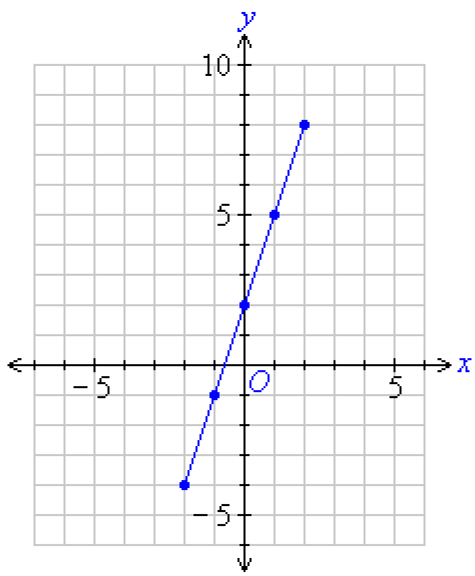
When $x = 1,\ y = 3 \times 1 + 2$
$= 3 + 2$
$= 5$

When $x = 2,\ y = 3 \times 2 + 2$
$= 6 + 2$
$= 8$

The table of values obtained after entering the values of *y* is as follows:

x	-2	-1	0	1	2
y	-4	-1	2	5	8

Draw a Cartesian plane and plot the points. Then join the points with a ruler to obtain a straight line graph.



Setting out:

Often, we set out the solution as follows.

$y = 3x + 2$

When $x = -2$, $y = 3(-2) + 2$
$= -6 + 2$
$= -4$

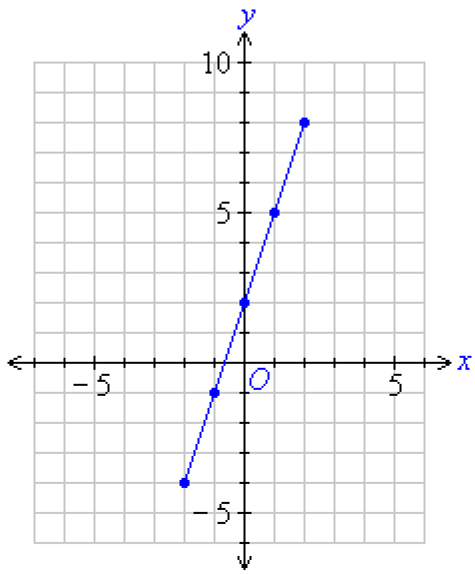
When $x = -1$, $y = 3(-1) + 2$
$= -3 + 2$
$= -1$

When $x = 0$, $y = 3 \times 0 + 2$
$= 0 + 2$
$= 2$

When $x = 1$, $y = 3 \times 1 + 2$
$= 3 + 2$
$= 5$

When $x = 2$, $y = 3 \times 2 + 2$
$= 6 + 2$
$= 8$

x	-2	-1	0	1	2
y	-4	-1	2	5	8



Example 7

Plot the graph of $y = -2x + 4$.

Solution:

$y = -2x + 4$

$$\text{When } x = -2,\ y = -2(-2) + 4$$
$$= 4 + 4$$
$$= 8$$

$$\text{When } x = -1,\ y = -2(-1) + 4$$
$$= 2 + 4$$
$$= 6$$

$$\text{When } x = 0,\ y = -2 \times 0 + 4$$
$$= 0 + 4$$
$$= 4$$

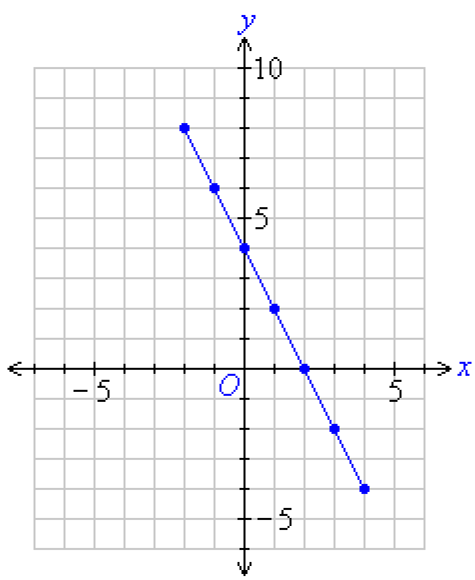
$$\text{When } x = 1,\ y = -2(1) + 4$$
$$= -2 + 4$$
$$= 2$$

$$\text{When } x = 2,\ y = -2(2)+4$$
$$= -4+4$$
$$= 0$$

$$\text{When } x = 3,\ y = -2(3)+4$$
$$= -6+4$$
$$= -2$$

$$\text{When } x = 4,\ y = -2(4)+4$$
$$= -8+4$$
$$= -4$$

x	-2	-1	0	1	2	3	4
y	8	6	4	2	0	-2	-4



Learning Objectives

In this section you will:

- Plot ordered pairs in a Cartesian coordinate system.
- Graph equations by plotting points.
- Graph equations with a graphing utility.
- Find -intercepts and -intercepts.
- Use the distance formula.

- Use the midpoint formula.

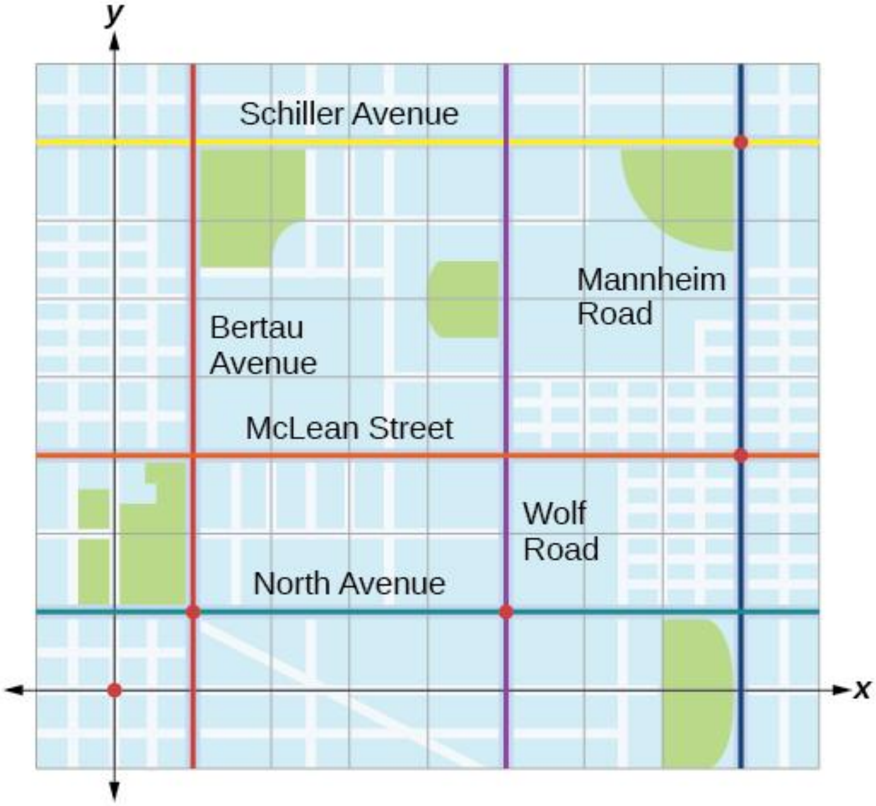


Figure 1.

Tracie set out from Elmhurst, IL, to go to Franklin Park. On the way, she made a few stops to do errands. Each stop is indicated by a red dot in (Figure). Laying a rectangular coordinate grid over the map, we can see that each stop aligns with an intersection of grid lines. In this section, we will learn how to use grid lines to describe locations and changes in locations.

Plotting Ordered Pairs in the Cartesian Coordinate System

An old story describes how seventeenth-century philosopher/mathematician René Descartes invented the system that has become the foundation of algebra while sick in bed. According to the story, Descartes was staring at a fly crawling on the ceiling when he realized that he could describe the fly's location in relation to the perpendicular lines formed by the adjacent walls of his room. He viewed the perpendicular lines as horizontal and vertical axes. Further, by dividing each axis into equal unit lengths, Descartes saw that it was possible to locate any object in a two-dimensional plane using just two numbers—the displacement from the horizontal axis and the displacement from the vertical axis.

While there is evidence that ideas similar to Descartes' grid system existed centuries earlier, it was Descartes who introduced the components that comprise the Cartesian coordinate system, a grid system having perpendicular axes. Descartes named the horizontal axis the *x*-axis and the vertical axis the *y*-axis.

The Cartesian coordinate system, also called the rectangular coordinate system, is based on a two-dimensional plane consisting of the *x*-axis and the *y*-axis. Perpendicular to each other, the axes divide the plane into four sections. Each section is called a quadrant; the quadrants are numbered counterclockwise as shown in (Figure)

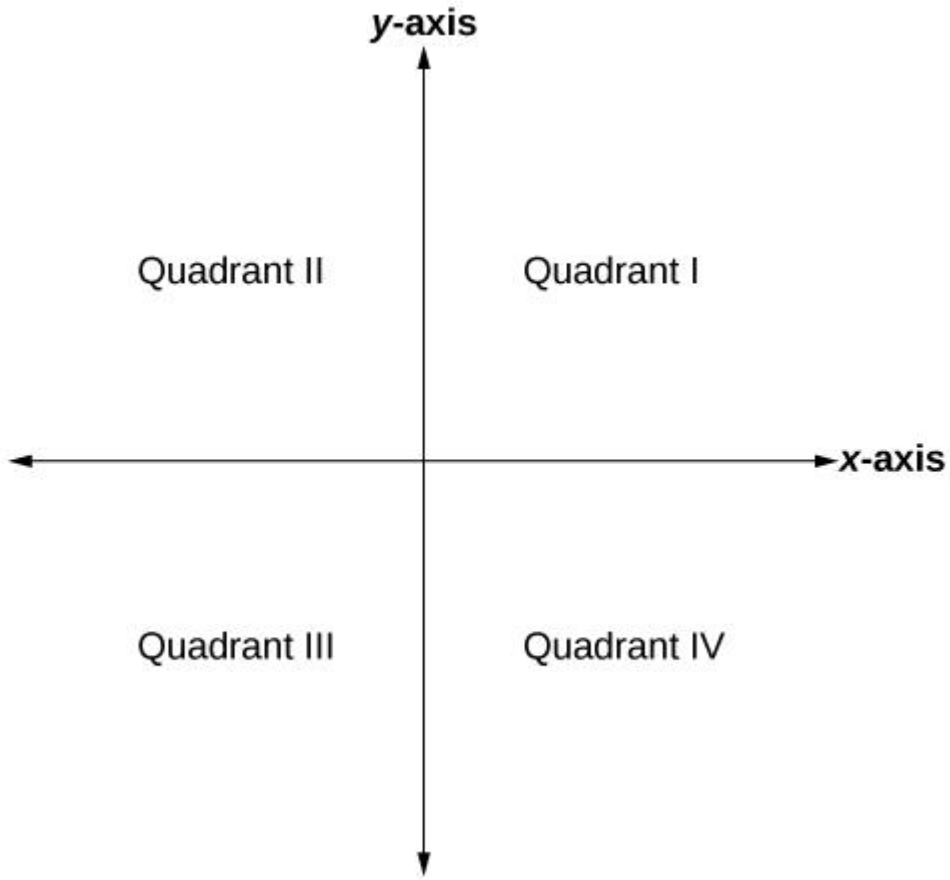


Figure 2.

The center of the plane is the point at which the two axes cross. It is known as the origin, or point From the origin, each axis is further divided into equal units: increasing, positive numbers to the right on the *x*-axis and up the *y*-axis; decreasing, negative numbers to the left on the *x*-axis and down the *y*-axis. The axes extend to positive and negative infinity as shown by the arrowheads in (Figure).

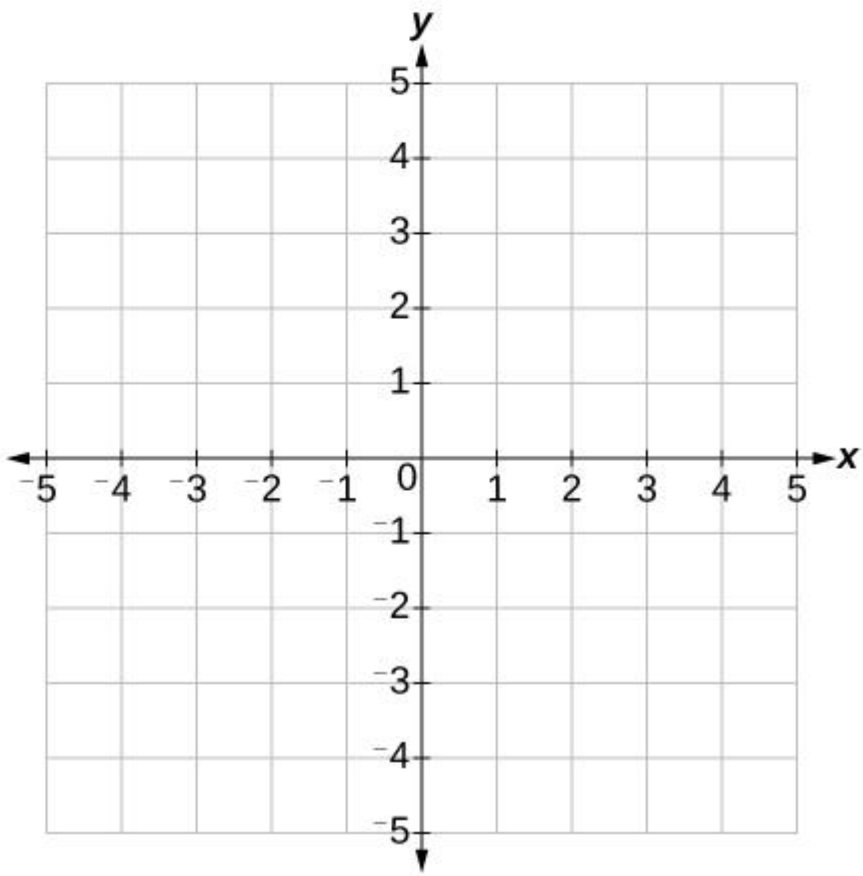


Figure 3.

Each point in the plane is identified by its *x*-coordinate, or horizontal displacement from the origin, and its *y*-coordinate, or vertical displacement from the origin. Together, we write them as an ordered pair indicating the combined distance from the origin in the form An ordered pair is also known as a coordinate pair because it consists of *x*- and *y*-coordinates. For example, we can represent the point in the plane by moving three units to the right of the origin in the horizontal direction, and one unit down in the vertical direction. See (Figure).

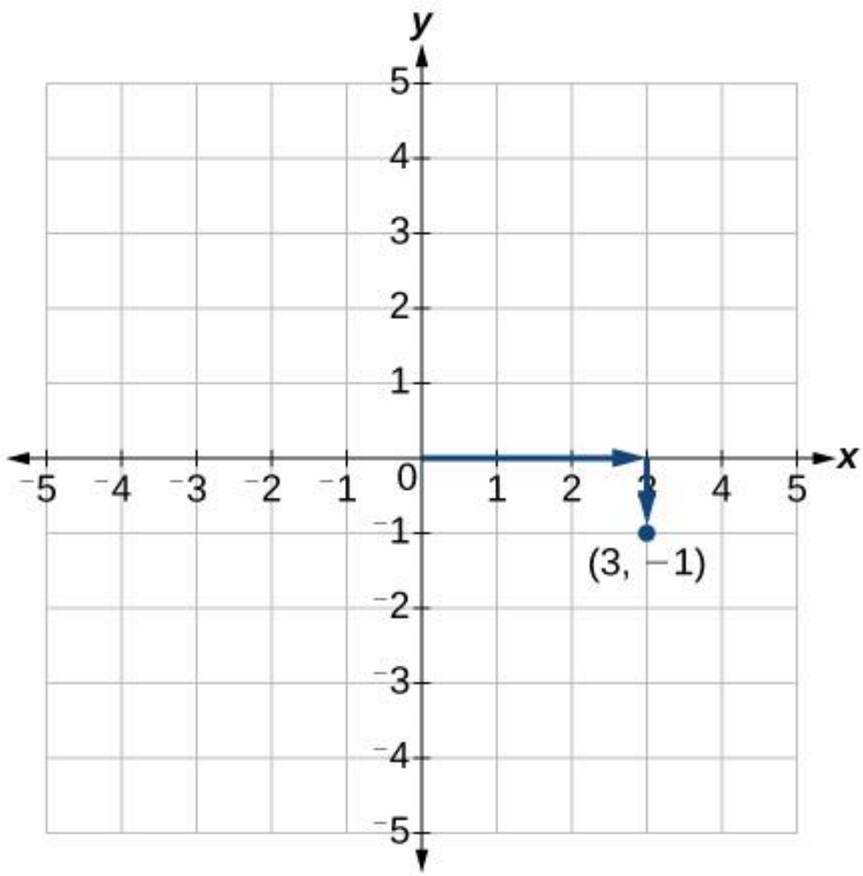


Figure 4.

When dividing the axes into equally spaced increments, note that the *x*-axis may be considered separately from the *y*-axis. In other words, while the *x*-axis may be divided and labeled according to consecutive integers, the *y*-axis may be divided and labeled by increments of 2, or 10, or 100. In fact, the axes may represent other units, such as years against the balance in a savings account, or quantity against cost, and so on. Consider the rectangular coordinate system primarily as a method for showing the relationship between two quantities.

Cartesian Coordinate System

A two-dimensional plane where the

- *x*-axis is the horizontal axis
- *y*-axis is the vertical axis

A point in the plane is defined as an ordered pair, such that *x* is determined by its horizontal distance from the origin and *y* is determined by its vertical distance from the origin.

Graphing Equations by Plotting Points

We can plot a set of points to represent an equation. When such an equation contains both an *x* variable and a *y* variable, it is called an equation in two variables. Its graph is called a graph in two variables. Any graph on a two-dimensional plane is a graph in two variables.

Suppose we want to graph the equation We can begin by substituting a value for *x* into the equation and determining the resulting value of *y*. Each pair of *x*– and *y*-values is an ordered pair that can be plotted. (Figure) lists values of *x* from –3 to 3 and the resulting values for *y*.

We can plot the points in the table. The points for this particular equation form a line, so we can connect them. See (Figure)**.** This is not true for all equations.

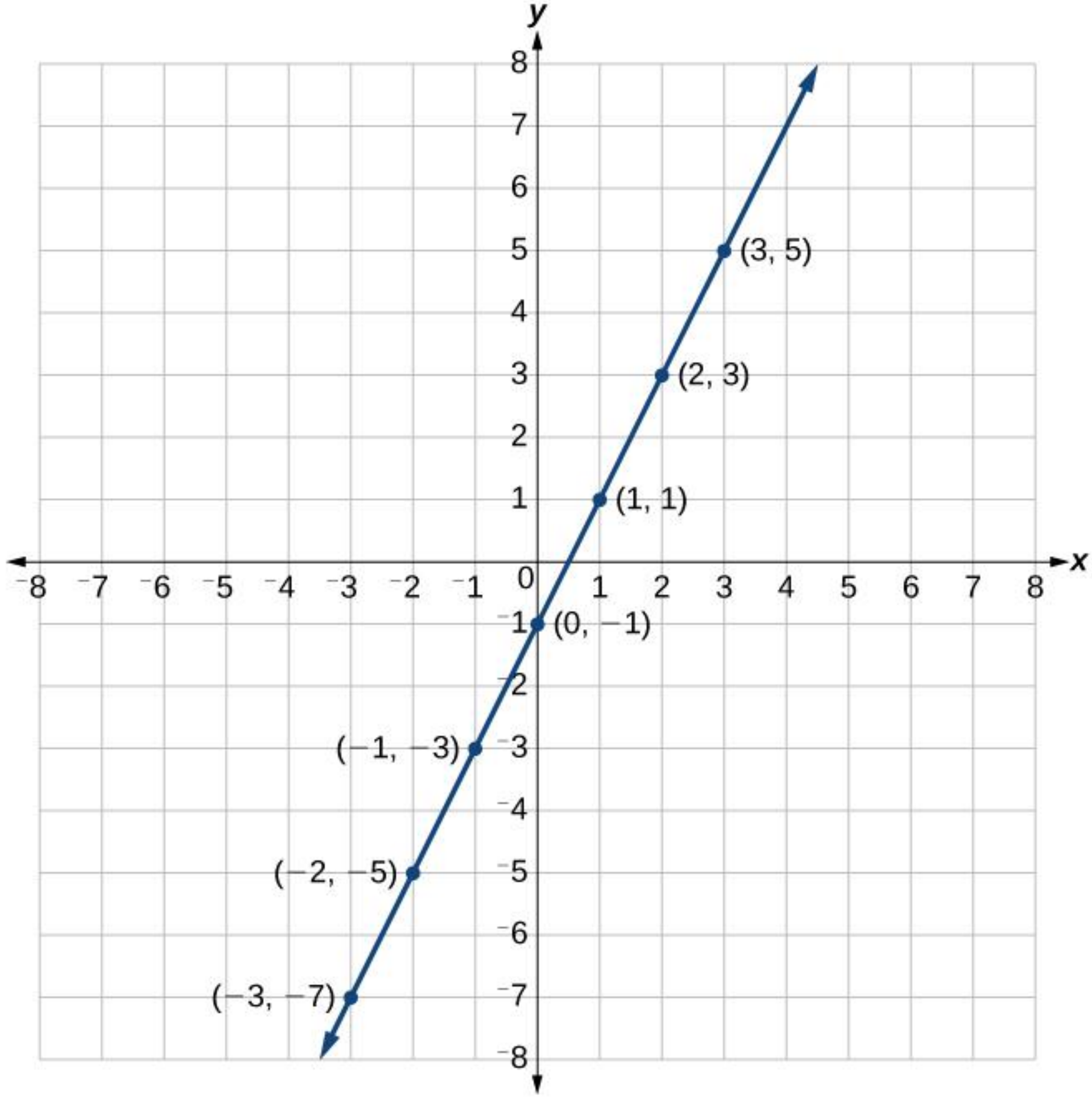


Figure 5.

Note that the *x*-values chosen are arbitrary, regardless of the type of equation we are graphing. Of course, some situations may require particular values of *x* to be plotted in order to see a particular result. Otherwise, it is logical to choose values that can be calculated easily, and it is always a good idea to choose values that are both negative and positive. There is no rule dictating how many points to plot, although we need at least two to graph a line. Keep in mind, however, that the more points we plot, the more accurately we can sketch the graph.

How To

Given an equation, graph by plotting points.

1. Make a table with one column labeled *x*, a second column labeled with the equation, and a third column listing the resulting ordered pairs.
2. Enter *x*-values down the first column using positive and negative values. Selecting the *x*-values in numerical order will make the graphing simpler.
3. Select *x*-values that will yield *y*-values with little effort, preferably ones that can be calculated mentally.
4. Plot the ordered pairs.
5. Connect the points if they form a line.

Graphing an Equation in Two Variables by Plotting Points

Graph the equation by plotting points.

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First, we construct a table similar to (Figure). Choose *x* values and calculate *y*.

Now, plot the points. Connect them if they form a line. See (Figure)

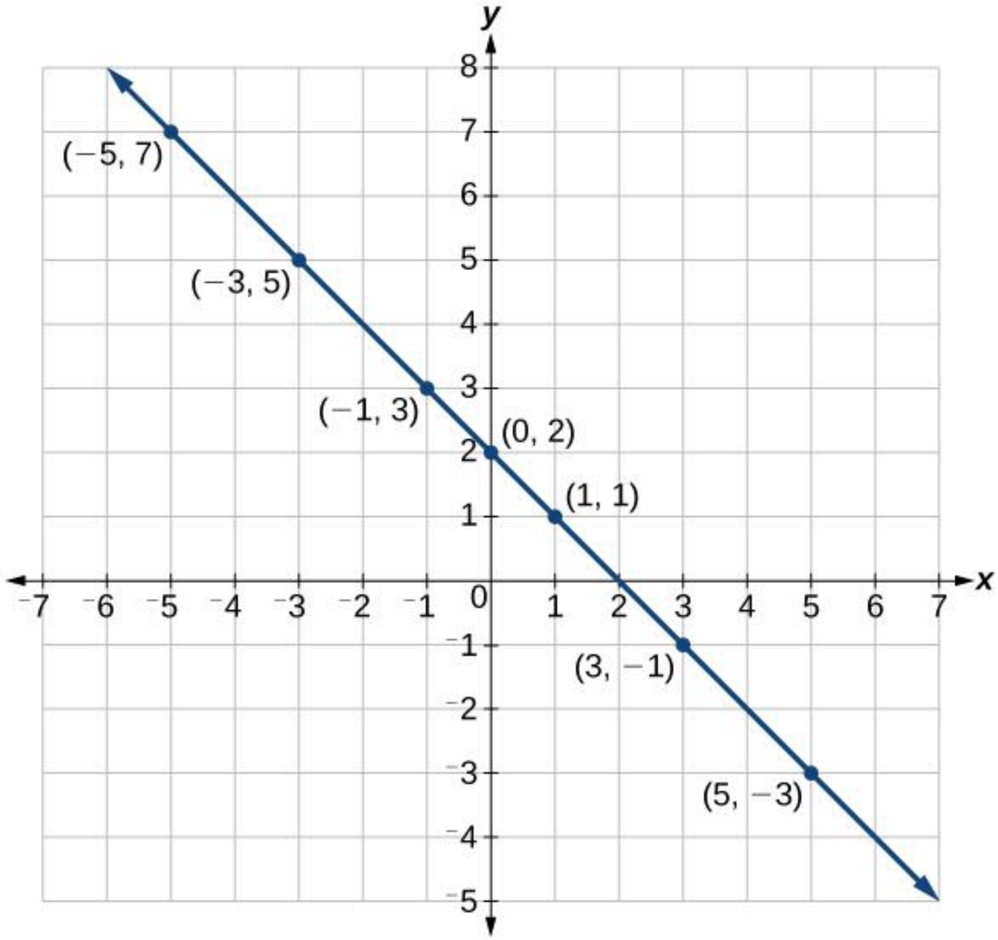

Figure 6.

[/hidden-answer]

Try It

Construct a table and graph the equation by plotting points:

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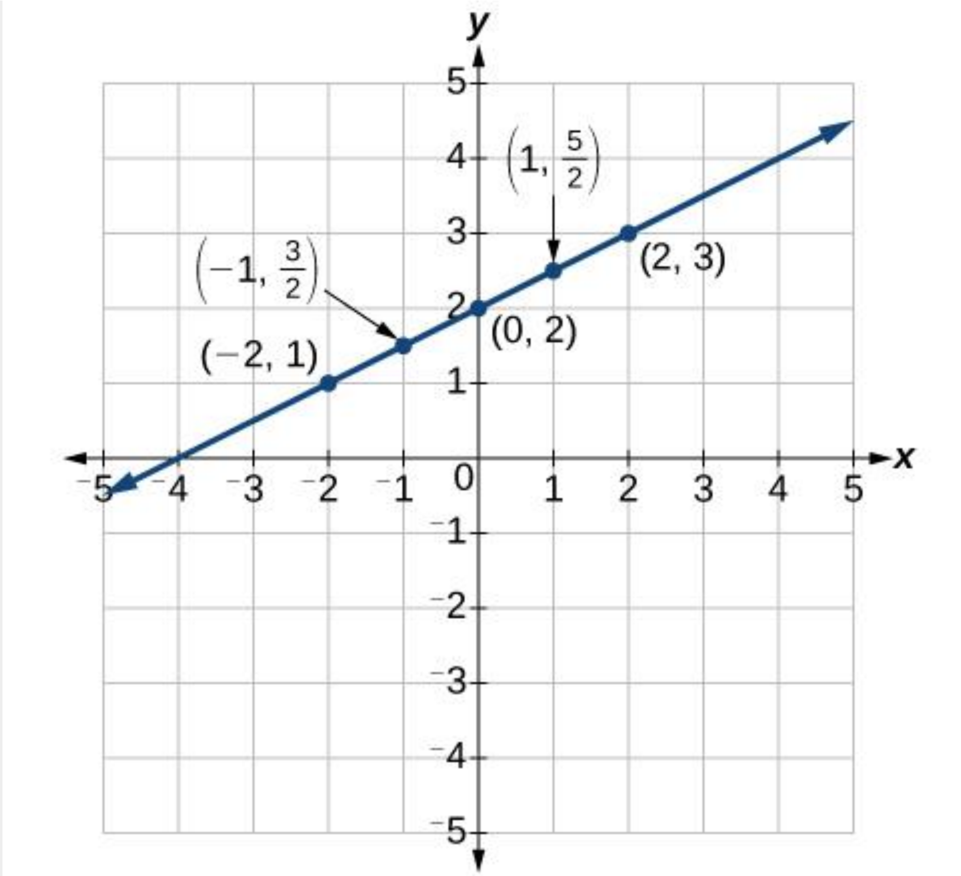

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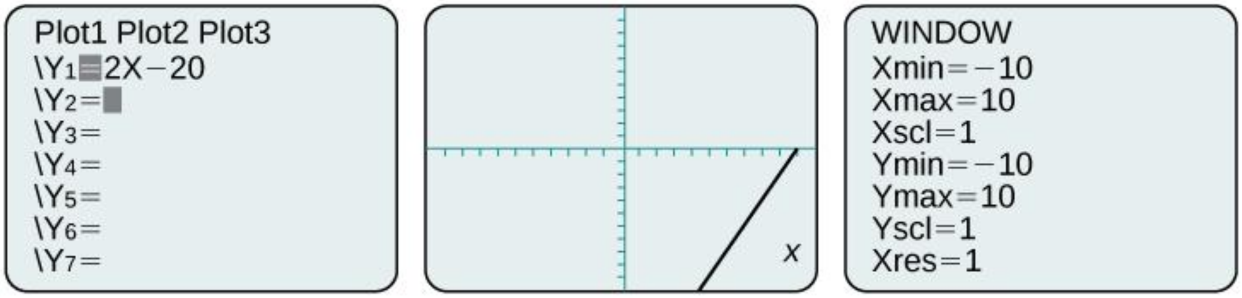
Graphing Equations with a Graphing Utility

Most graphing calculators require similar techniques to graph an equation. The equations sometimes have to be manipulated so they are written in the style =_____. The TI-84 Plus, and many other calculator makes and models, have a mode function, which allows the window (the screen for viewing the graph) to be altered so the pertinent parts of a graph can be seen.

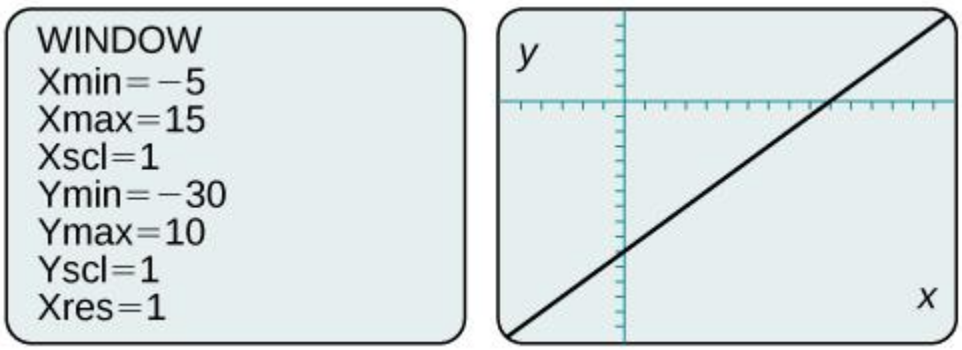
For example, the equation has been entered in the TI-84 Plus shown in (Figure)**a.** In (Figure)**b,** the resulting graph is shown. Notice that we cannot see on the screen where the graph crosses the axes. The standard window screen on the TI-84 Plus shows and See (Figure)**c**.



(a) (b) (c)

Figure 7. a. Enter the equation. b. This is the graph in the original window. c. These are the original settings.

By changing the window to show more of the positive *x*-axis and more of the negative *y*-axis, we have a much better view of the graph and the *x*- and *y*-intercepts. See (Figure)**a** and (Figure)**b.**



(a) (b)

Figure 8. a. This screen shows the new window settings. b. We can clearly view the intercepts in the new window.

Using a Graphing Utility to Graph an Equation

Use a graphing utility to graph the equation:

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Enter the equation in the *y*= function of the calculator. Set the window settings so that both the *x*- and *y*- intercepts are showing in the window. See (Figure).

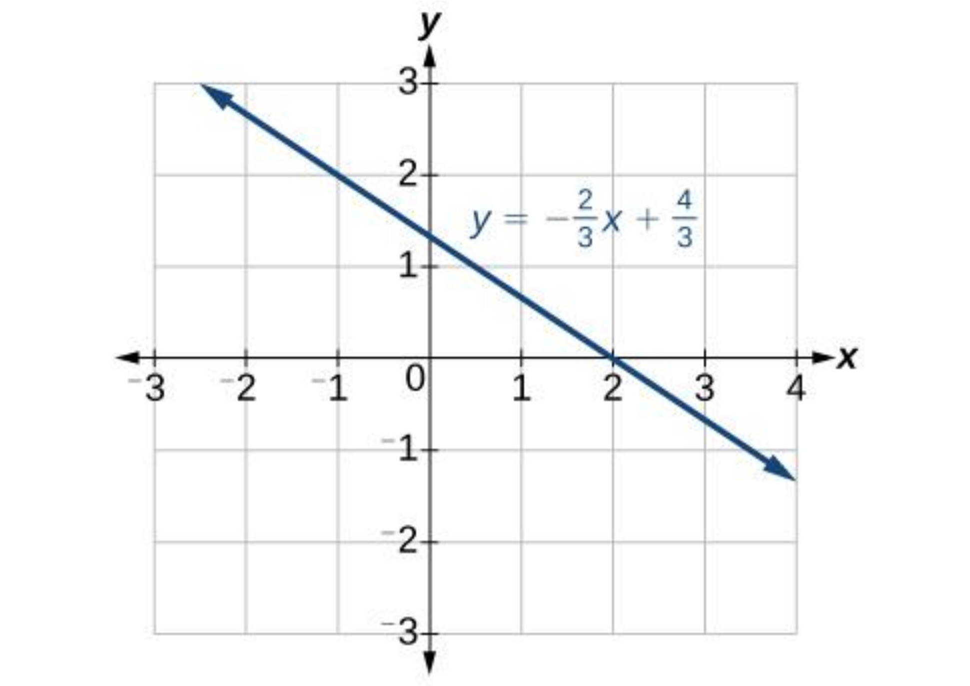

Figure 9.

[/hidden-answer]

Finding *x*-intercepts and *y*-intercepts

The intercepts of a graph are points at which the graph crosses the axes. The *x*-intercept is the point at which the graph crosses the *x*-axis. At this point, the *y*-coordinate is zero. The *y*-intercept is the point at which the graph crosses the *y*-axis. At this point, the *x*-coordinate is zero.

To determine the *x*-intercept, we set *y* equal to zero and solve for *x*. Similarly, to determine the *y*-intercept, we set *x* equal to zero and solve for *y*. For example, lets find the intercepts of the equation

We can confirm that our results make sense by observing a graph of the equation as in (Figure). Notice that the graph crosses the axes where we predicted it would.

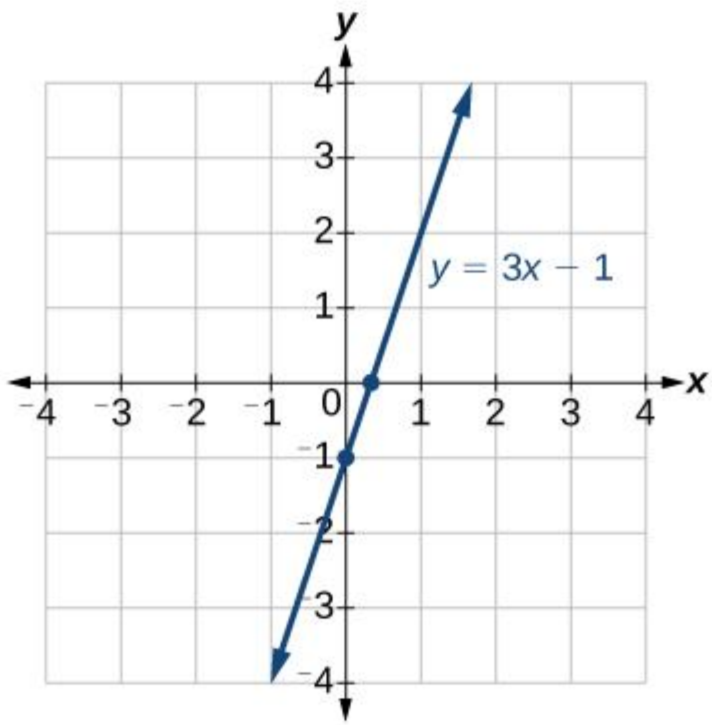

Figure 10.

Using the Distance Formula

Derived from the Pythagorean Theorem, the distance formula is used to find the distance between two points in the plane. The Pythagorean Theorem, is based on a right triangle where *a* and *b* are the lengths of the legs adjacent to the right angle, and *c* is the length of the hypotenuse. See [(Figure)].

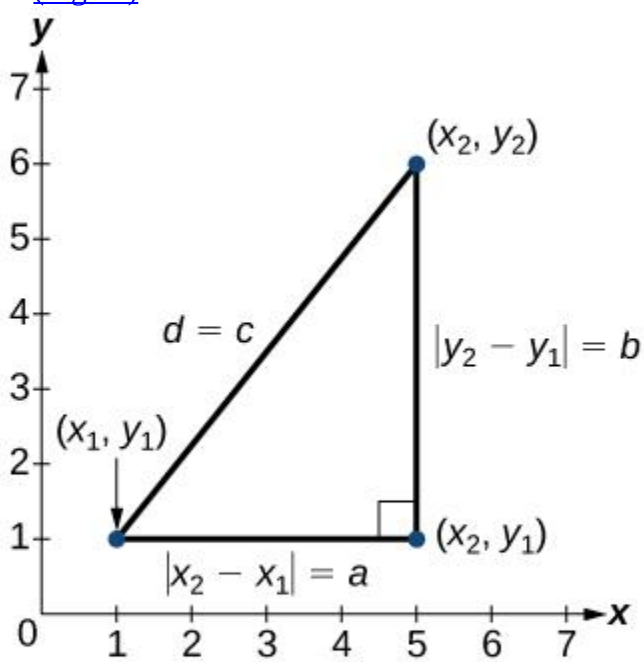

Figure 12.

The relationship of side sand to side *d* is the same as that of sides *a* and *b* to side *c*. We use the absolute value symbol to indicate that the length is a positive number because the absolute value of any number is positive. (For example,) The symbols and indicate that the lengths of the sides of the triangle are positive. To find the length *c*, take the square root of both sides of the Pythagorean Theorem.

It follows that the distance formula is given as

We do not have to use the absolute value symbols in this definition because any number squared is positive.

The Distance Formula

Given endpoints and the distance between two points is given by

Finding the Distance between Two Points

Find the distance between the points and

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[hidden-answer a="fs-id3223093″]

Let us first look at the graph of the two points. Connect the points to form a right triangle as in (Figure).

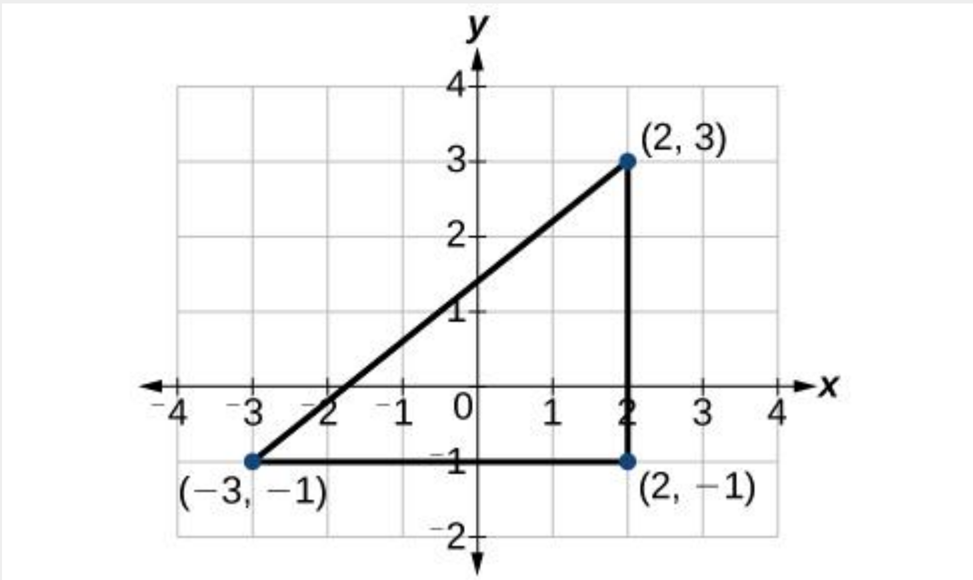

Figure 13.

Then, calculate the length of *d* using the distance formula.

Find the distance between two points: and

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[hidden-answer a=”fs-id2667607″]

[/hidden-answer]

Finding the Distance between Two Locations

Let’s return to the situation introduced at the beginning of this section.

Tracie set out from Elmhurst, IL, to go to Franklin Park. On the way, she made a few stops to do errands. Each stop is indicated by a red dot in (Figure). Find the total distance that Tracie traveled. Compare this with the distance between her starting and final positions.

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[hidden-answer a=”fs-id2522863″]

The first thing we should do is identify ordered pairs to describe each position. If we set the starting position at the origin, we can identify each of the other points by counting units east (right) and north (up) on the grid. For example, the first stop is 1 block east and 1 block north, so it is at The next stop is 5 blocks to the east, so it is at After that, she traveled 3 blocks east and 2 blocks north to Lastly, she traveled 4 blocks north to We can label these points on the grid as in (Figure).

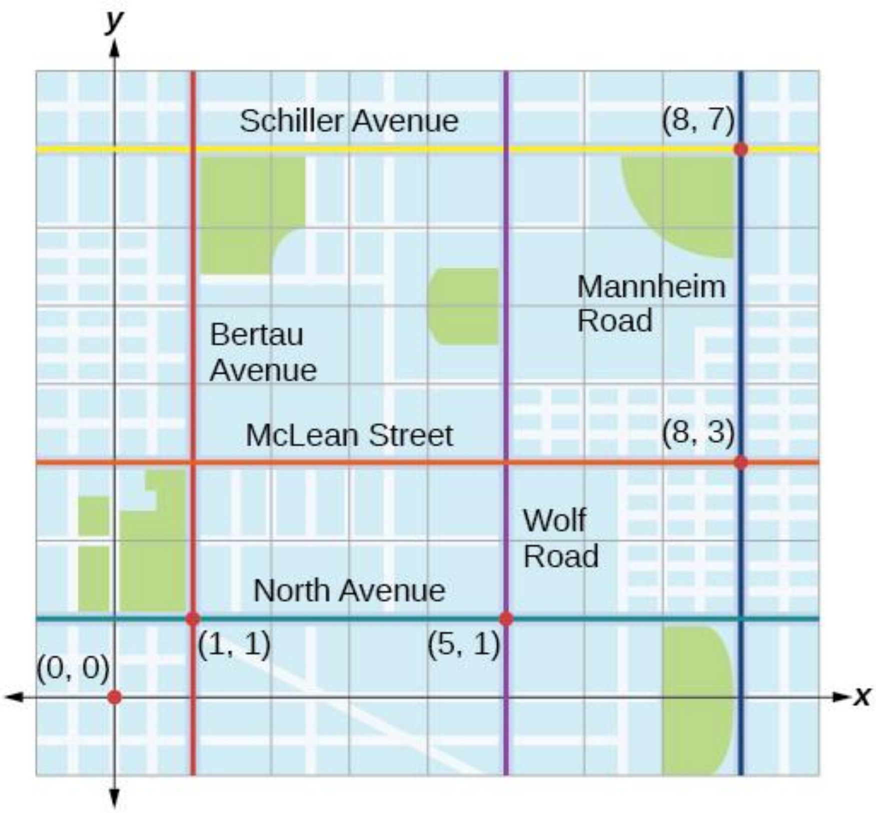

Figure 14.

Next, we can calculate the distance. Note that each grid unit represents 1,000 feet.

- From her starting location to her first stop at Tracie might have driven north 1,000 feet and then east 1,000 feet, or vice versa. Either way, she drove 2,000 feet to her first stop.
- Her second stop is at So from to Tracie drove east 4,000 feet.
- Her third stop is at There are a number of routes from to Whatever route Tracie decided to use, the distance is the same, as there are no angular streets between the two points. Let's say she drove east 3,000 feet and then north 2,000 feet for a total of 5,000 feet.
- Tracie's final stop is at This is a straight drive north from for a total of 4,000 feet.

Next, we will add the distances listed in (Figure).

From/To	**Number of Feet Driven**
to	2,000

From/To	Number of Feet Driven
to	4,000
to	5,000
to	4,000
Total	15,000

The total distance Tracie drove is 15,000 feet, or 2.84 miles. This is not, however, the actual distance between her starting and ending positions. To find this distance, we can use the distance formula between the points and

At 1,000 feet per grid unit, the distance between Elmhurst, IL, to Franklin Park is 10,630.14 feet, or 2.01 miles. The distance formula results in a shorter calculation because it is based on the hypotenuse of a right triangle, a straight diagonal from the origin to the point Perhaps you have heard the saying "as the crow flies," which means the shortest distance between two points because a crow can fly in a straight line even though a person on the ground has to travel a longer distance on existing roadways.[/hidden-answer]

Using the Midpoint Formula

When the endpoints of a line segment are known, we can find the point midway between them. This point is known as the midpoint and the formula is known as the midpoint formula. Given the endpoints of a line segment, and the midpoint formula states how to find the coordinates of the midpoint

A graphical view of a midpoint is shown in (Figure). Notice that the line segments on either side of the midpoint are congruent.

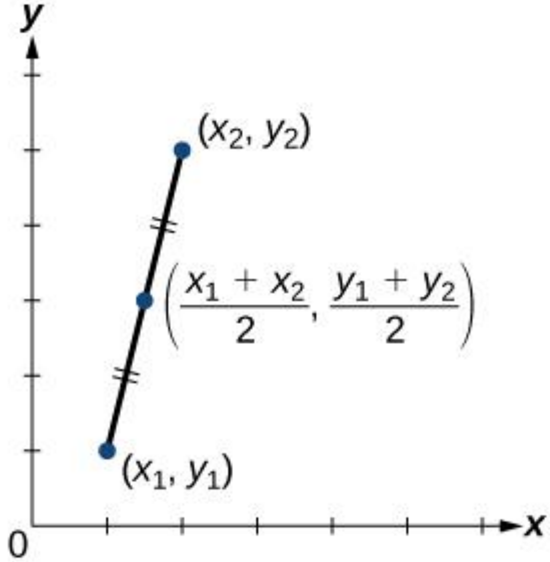

Figure 15.

Finding the Midpoint of the Line Segment

Find the midpoint of the line segment with the endpoints and

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Use the formula to find the midpoint of the line segment.

[/hidden-answer]

Try It

Find the midpoint of the line segment with endpoints and

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Frequency Distribution

Frequency

Frequency is how often something occurs.

Example: Sam played football on:

- Saturday Morning,
- Saturday Afternoon
- Thursday Afternoon

The frequency was 2 on Saturday, 1 on Thursday and 3 for the whole week.

Frequency Distribution

By counting frequencies we can make a **Frequency Distribution** table.

Example: Goals

Sam's team has scored the following numbers of goals in recent games

2, 3, 1, 2, 1, 3, 2, 3, 4, 5, 4, 2, 2, 3

Sam put the numbers in order, then added up:

- how often 1 occurs (2 times),
- how often 2 occurs (5 times),
- etc,

and wrote them down as a **Frequency Distribution** table.

From the table we can see interesting things such as

- getting 2 goals happens most often
- only once did they get 5 goals

This is the definition:

Frequency Distribution: values and their frequency (how often each value occurs).

Here is another example:

Example: Newspapers

These are the numbers of newspapers sold at a local shop over the last 10 days:

22, 20, 18, 23, 20, 25, 22, 20, 18, 20

Let us count how many of each number there is:

Papers Sold	Frequency
18	2
19	0
20	4
21	0
22	2
23	1
24	0
25	1

It is also possible to **group** the values. Here they are grouped in 5s:

Papers Sold	Frequency
15-19	2
20-24	7

25-29 1

What is a Frequency Distribution Table?
Frequency tells you **how often something happened**. The frequency of an observation tells you the number of times the observation occurs in the data. For example, in the following list of numbers, the frequency of the number 9 is 5 (because it occurs 5 times):
1, 2, 3, 4, 6, 9, 9, 8, 5, 1, 1, 9, 9, 0, 6, 9.

Tables can show either categorical variables (sometimes called qualitative variables) or quantitative variables (sometimes called numeric variables). You can think of categorical variables as categories (like eye color or brand of dog food) and quantitative variables as numbers.
If you aren't quite sure of the difference, see: Qualitative or quantitative? How to tell.
The following table shows what family planning methods were used by teens in Kweneng, West Botswana. The left column shows the **categorical variable** (Method) and the right column is the **frequency** — the number of teens using that particular method (image courtesy of KSU).

Method	Number
Abstinence	14
Condoms	47
Injectables	1
Norplant	1
Pill	35
None	307
Total	405

A frequency distribution table showing categorical variables.

Frequency distribution tables give you a **snapshot** of the data to allow you to find patterns. A quick look at the above frequency distribution table tells you the majority of teens don't use any birth control at all.
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How to make a Frequency Distribution Table: Examples

Example 1
Tally marks are often used to make a frequency distribution table. For example, let's say you survey a number of households and find out how many pets they own. The results are 3, 0, 1, 4, 4, 1, 2, 0, 2, 2, 0, 2, 0, 1, 3, 1, 2, 1, 1, 3. Looking at that string of numbers boggles the eye; a frequency distribution table will make the data easier to understand.

Steps
To make the frequency distribution table, first **write the categories** in one column (number of pets):

Number of Pets (x)	Tally	Frequency (f)
0		
1		
2		
3		
4		

Next, **tally the numbers** in each category (from the results above). For example, the number zero appears four times in the list, so put four tally marks "||||":

Number of Pets (x)	Tally	Frequency (f)
0	||||	
1	~~||||~~ |	
2	~~||||~~	
3	|||	
4	||	

Finally, count up the tally marks and **write the frequency** in the final column. The frequency is just the total. You have four tally marks for "0", so put 4 in the last column:

Number of Pets (x)	Tally	Frequency (f)
0	||||	4
1	~~||||~~ |	6
2	~~||||~~	5
3	|||	3
4	||	2

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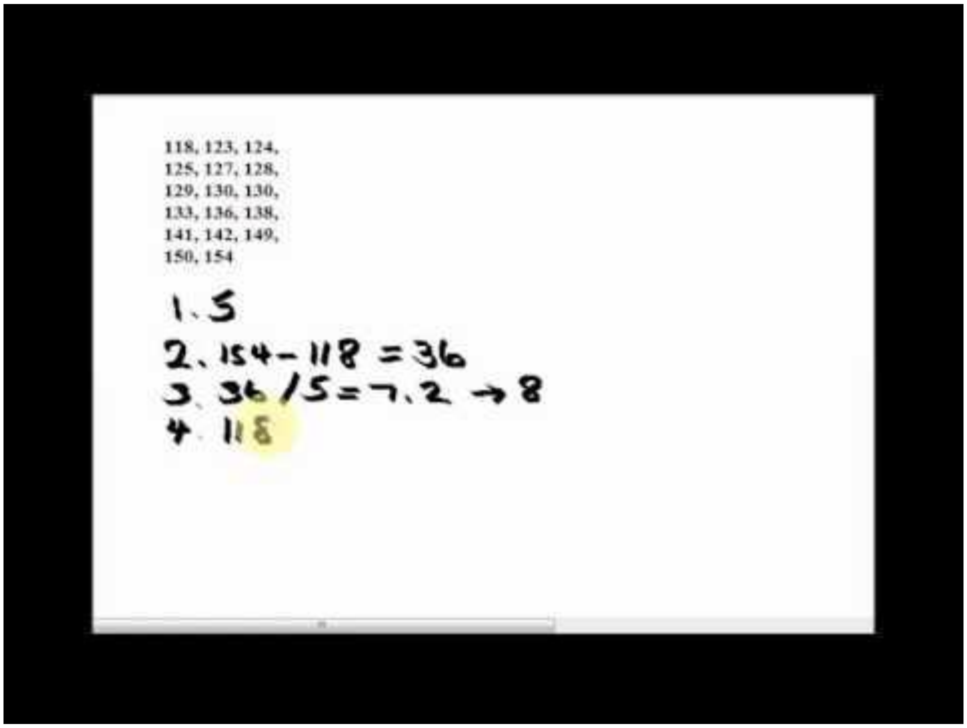
Example 2
Watch the video or read the steps below:



How to Draw a Frequency Distribution Table (Slightly More Complicated Example)
A **frequency distribution table** is one way you can organize data so that it makes more sense. For example, let's say you have a list of IQ scores for a gifted classroom in a particular elementary school. The **IQ scores** are: 118, 123, 124, 125, 127, 128, 129, 130, 130, 133, 136, 138, 141, 142, 149, 150, 154. That list doesn't tell you much about anything. You could draw a **frequency distribution table**, which will give a better picture of your data than a simple list.

How to Draw a Frequency Distribution Table: Steps.

Part 1: Choosing Classes
Step 1: Figure out how many classes (categories) you need. There are no hard rules about how many classes to pick, but there are a couple of general guidelines:

- Pick between 5 and 20 classes. For the list of IQs above, we picked 5 classes.
- Make sure you have a few items in each category. For example, if you have 20 items, choose 5 classes (4 items per category), not 20 classes (which would give you only 1 item per category).

Note: There is a more mathematical way to choose classes. The formula is log(observations)\log(2). You would round up the answer to the next integer. For example, log17\log2 = 4.1 will be rounded up to become 5. (Thank you to AymanMasry for that tip).

Part 2: Sorting the Data
Step 2: Subtract the minimum data value from the maximum data value. For example, our IQ list above had a minimum value of 118 and a maximum value of 154, so:
154 – 118 = **36**

Step 3: Divide your answer in Step 2 by the number of classes you chose in Step 1.
36 / 5 = **7.2**
Step 4: Round the number from Step 3 up to a whole number to get the class width. Rounded up, 7.2 becomes **8**.
Step 5: Write down your lowest value for your first minimum data value:
The lowest value is **118**
Step 6: Add the class width from Step 4 to Step 5 to get the next lower class limit:
118 + 8 = **126**
Step 7: Repeat Step 6 for the other minimum data values (in other words, keep on adding your class width to your minimum data values) until you have created the number of classes you chose in Step 1. We chose 5 classes, so our 5 minimum data values are:
118
126 (118 + 8)
134 (126 + 8)
142 (134 + 8)
150 (142 + 8)
Step 8: Write down the upper class limits. These are the highest values that can be in the category, so in most cases you can subtract 1 from the class width and add that to the minimum data value. For example:
118 + (8 – 1) = 125
118 – 125
126 – 133
134 – 141
142 – 149
150 – 157

3. Finishing the Table Up
Step 9: Add a second column for the number of items in each class, and label the columns with appropriate headings:

IQ	NUMBER
118-125	
126-133	
134-141	
142-149	
150-157	

Step 10: Count the number of items in each class, and put the total in the second column. The list of IQ scores are: 118, 123, 124, 125, 127, 128, 129, 130, 130, 133, 136, 138, 141, 142, 149, 150, 154.

IQ	NUMBER
118-125	4
126-133	6
134-141	3
142-149	2
150-157	2

That's How to Draw a Frequency Distribution Table, the easy way!

Like the explanation? Check out our statistics how-to book, with hundreds more step by step solutions, just like this one!

Tip: If you are working with large numbers (like hundreds or thousands), round Step 4 up to a large whole number that's easy to make into classes, like 100, 1000, or 10,000. Likewise with very small numbers — you may want to round to 0.1, 0.001 or a similar division.

Frequency Distribution and Graphs

1. 1. Chapter 2
Frequency Distributions
and Graphs
2. 2. A frequency distribution is the organization of raw data in table from, using classes and frequency.
3. 3. The number of miles that the employees of a large department store traveled to work each day
4. 4. How to construct a grouped frequency Distribution?
5. 5. Number of classes
It should be between 5 and 20.
Some Statisticians use " 2k " rule.
6. 6. 2 to k rule
Essentially we would look to construct k classes for our frequency distribution, when the value of 2k first exceeds the number of observations in our sample. So, if we had a sample with 39 observations, we would first consider constructing 6 classes, because 26 = 64, the first power of 2 with a value larger than the sample size of 39.
7. 7. A guide, not a dictator.
Strictly speaking the 2k rule is a guide, not a rule. If the 2k rule suggests you need 6 classes, also consider using 5 or 7 classes ... but certainly not 3 or 9.
8. 8. Class interval or class width H : the highest value, L: the smallest value
Class interval can also be estimated based on # of

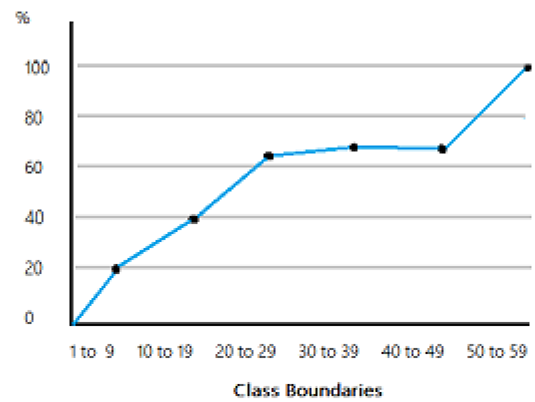
observations
Select the lower limit of the first class and set the limits of each class
It could be L or any value smaller than L.
It should be an even multiple of the class interval.

9. 9. There should be between 5 and 20 classes.
10. 10. The classes must be continuous.
11. 11. The classes must be exhaustive.
12. 12. The classes must be mutually exclusive.
13. 13. The classes must be equal in width.
Relative frequency
Relative frequency of a class is the frequency of that class divided by to total number of frequency.
Example
These data represent the record high temperatures for each of the 50 states. Construct a grouped frequency distribution for the data using 7 classes.
14. 14.
15. 15. Histogram
A histogram is a graph that displays the data by using contiguous vertical bars (unless the frequency of a class is 0) of various heights to represent the frequencies of the classes.
16. 16. Example
Construct a histogram to represent the data shown below for the record high temperature:
17. 17. 18
Histogram
15
12
9
6
3
99.5
109.5
104.5
124.5
119.5
114.5
129.5
The largest concentration is in the class 109.5 – 114.5.
18. 18. 18
Frequency Polygone
15
12
9
6
3
99.5
109.5
104.5
124.5
119.5
114.5
129.5
19. 19. The Ogive is a graph that represents the cumulative frequencies for the classes in a frequency distribution.
20. 20.
21. 21. Cumulative Frequency Polygone
50
40
30
20
10
99.5
109.5
104.5
124.5
119.5
114.5
129.5
22. 22. Other types of Graphs
Bar Chart
Bar Chart is use to represent a frequency distribution for a categorical variable, and the frequencies are displayed by the heights of vertical bars.
23. 23. Example
The table shown here displays the number of crimes investigated by law enforcement officers in U.S. national parks during 1995. Construct a Bar chart for the data.
24. 24. 164
150
100
50
34
29
13
Homicide
Assault
Rape
Robbery
Total number of crime: 234
25. 25. Pie Graph
 A pie graph is a circle that is divided into sections or wedges according to the percentage of frequencies in each category of the distribution.
26. 26. Example
This frequency distribution shows the number of pounds of each snack food eaten during the 1998 Super Bowl. Construct a pie graph for the data.
27. 27. We need to find percentages for each category and then compute the corresponding sectors so that we divide the circle proportionally.
28. 28.
29. 29. Stem and Leaf Plots
A stem and leaf plot is a data plot that uses part of the data value as the stem and part of the data value as the leaf to form groups or classes.

30. 30. Example
At an outpatient testing center, the number of cardiograms performed each day for 20 days is shown. Construct a tem and leaf plot for the data.
31. 31. It is helpful to arrange the data in order but it is not required.
02, 13, 14, 20, 23, 25, 31, 32, 32, 32, 32, 33, 36, 43, 44, 44, 45, 51, 52, 57
32. 32. EXERCISES 1
The following data represent the color of men's dress shirts purchased in the men's department of a large department store. Construct a categorical frequency distribution, bar chart and pie chart for the data (W= white, BL= blue,
BR= brown, Y= yellow, G= gray).
33. 33. EXERCISES 1(Cont.)
34. 34. EXERCISES 2
The ages of the signers of the Declaration of Independence of the US are shown below.
35. 35. EXERCISES 2 (Cont.)
Construct a frequency distribution using seven classes. Include relative frequency, percentage and Cumulative frequency.
Construct a histogram, frequency poly-gone, and Ogive.
Develop a stem-and-leaf plot for the data.
36. 36. Thank You for your attention!
Good Luck!

Topic: Cumulative Frequency Distribution

What is an Ogive Graph?



An ogive (*oh-jive*), sometimes called a cumulative frequency polygon, is a type of frequency polygon that shows cumulative frequencies. In other words, the cumulative percents are added on the graph from left to right.

An ogive graph plots **cumulative frequency** on the y-axis and **class boundaries** along the x-axis. It's very similar to a histogram, only instead of rectangles, an ogive has a single point marking where the top right of the rectangle would be. It is usually easier to create this kind of graph from a frequency table.

How to Draw an Ogive Graph

Example question: Draw an Ogive graph for the following set of data:
02, 07, 16, 21, 31, 03, 08, 17, 21, 55
03, 13, 18, 22, 55, 04, 14, 19, 25, 57
06, 15, 20, 29, 58.

Step 1: Make a relative frequency table from the data. The first column has the class limits, the second column has the frequency (the count) and the third column has the relative frequency (class frequency / total number of items):

Class Limits	Frequency	Relative Frequency (# / Total)
01 to 09	5	5/25 = .20
10 to 19	5	5/25 = .20
20 to 29	6	6/25 = 0.24
30 to 39	1	1/25 =0.04
40 to 49	0	0/25 = 0
50 to 59	8	8/25 = 0.32

If you aren't sure how to create your class limits(also called bins), watch the video at the bottom of this article or see: What is a Bin in Statistics?

Step 2: Add a fourth column and cumulate (add up) the frequencies in column 2, going down from top to bottom. For example, the second entry is the sum of the first row and the second row in the frequency column (5 + 5 = 10), and the third entry is the sum of the first, second, and third rows in the frequency column (5 + 5 + 6 = 16):

Class Limits	Frequency	Relative Frequency (# / Total)	Cumulative Frequency
01 to 09	5	5/25 = .20	5
10 to 19	5	5/25 = .20	10
20 to 29	6	6/25 = 0.24	16
30 to 39	1	1/25 =0.04	17
40 to 49	0	0/25 = 0	17
50 to 59	8	8/25 = 0.32	25

Step 3: Add a fifth column and cumulate the **relative frequencies** from column 3. If you do this step correctly, your values should add up to 100% (or 1 as a decimal):

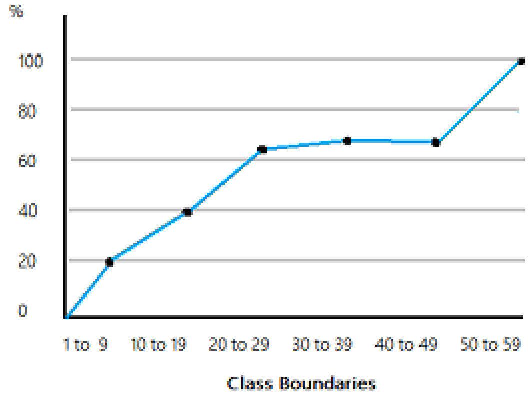
Class Limits	Frequency	Relative Frequency (# / Total)	Cumulative Frequency	Cumulative Rel . Freq.
01 to 09	5	5/25 = .20	5	0.2
10 to 19	5	5/25 = .20	10	0.4
20 to 29	6	6/25 = 0.24	16	0.64
30 to 39	1	1/25 =0.04	17	0.68
40 to 49	0	0/25 = 0	17	0.68
50 to 59	8	8/25 = 0.32	25	1

Step 4: Draw an x-y graph with percent cumulative relative frequency on the y-axis (from 0 to 100%, or as a decimal, 0 to 1). Mark the x-axis with the class boundaries.

Step 5: Plot your points. **Note**: Each point should be plotted on the upper limit of the class boundary. For example, if your first class boundary is 0 to 10, the point should be plotted at 10.

Step 6: Connect the dots with straight lines. theogive is one continuous line, made up of several smaller lines that connect pairs of dots, moving from left to right.

The finished graph for this sample data:



Topic: Measures of Central Tendency

Cumulative Frequency Curve

As we know, a picture speaks a thousand words. In the world of statics, graphs, in particular, are very important, as they help us make sense of the data and understand it better. So let us study the graphical representation of Cumulative Frequency Curve.

What is Cumulative Frequency?

The frequency is the number of times an event occurs within a given scenario. Cumulative frequency is defined as the running total of frequencies. It is the sum of all the previous frequencies up to the current point. It is easily understandable through a Cumulative Frequency Table.

Marks	Frequency (No. of Students)	Cumulative Frequency
0 – 5	2	2
5 – 10	10	12

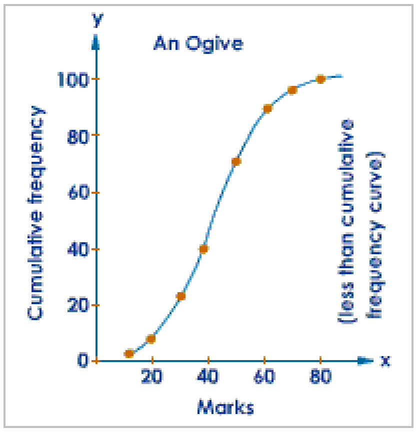
10 – 15	5	17
15 – 20	5	22

Cumulative Frequency is an important tool in Statistics to tabulate data in an organized manner. Whenever you wish to find out the popularity of a certain type of data, or the likelihood that a given event will fall within certain frequency distribution, a cumulative frequency table can be most useful. Say, for example, the Census department has collected data and wants to find out all residents in the city aged below 45. In this given case, a cumulative frequency table will be helpful.

Browse more Topics under Statistics

- Data
- Mean
- Median
- Mode
- Bar Graphs and Histogram
- Frequency Distribution
- Frequency Polygon
- Range and Mean Deviation
- Range and Mean Deviation for Grouped Data
- Range and Mean Deviation for Ungrouped Data
- Variance and Standard Deviation

Cumulative Frequency Curve



A curve that represents the cumulative frequency distribution of grouped data on a graph is called a Cumulative Frequency Curve or an Ogive. Representing cumulative frequency data on a graph is the most efficient way to understand the data and derive results.

Learn more about Frequency Polygon here.

There are two types of Cumulative Frequency Curves (or Ogives) :

- More than type Cumulative Frequency Curve
- Less than type Cumulative Frequency Curve

More Than Type Cumulative Frequency Curve

Here we use the lower limit of the classes to plot the curve.

How to plot a More than type Ogive:

1. In the graph, put the lower limit on the x-axis
2. Mark the cumulative frequency on the y-axis.
3. Plot the points (x,y) using lower limits (x) and their corresponding Cumulative frequency (y)
4. Join the points by a smooth freehand curve. It looks like an upside down *S*.

Learn more about Bar Graphs and Histogram here in detail.

Less than Type Cumulative Frequency Curve

Here we use the upper limit of the classes to plot the curve.

How to plot a Less than type Ogive:

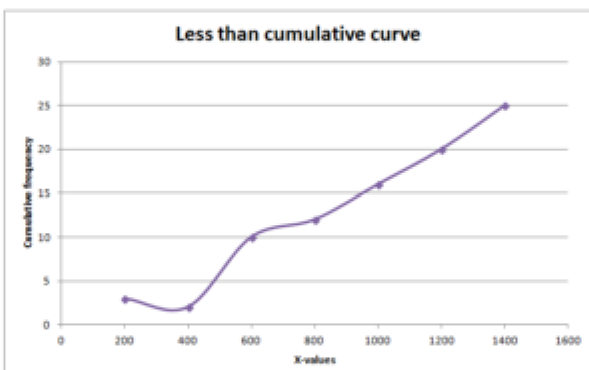
1. In the graph, put the upper limit on the x-axis
2. Mark the cumulative frequency on the y-axis.
3. Plot the points (x,y) using upper limits (x) and their corresponding Cumulative frequency (y)
4. Join the points by a smooth freehand curve. It looks like an elongated *S*.

Cumulative Graphs can also be used to calculate the Median of given data. If you draw both the curves on the same graph, the point at which they intersect, the corresponding value on the x-axis, represents the Median of the given data set.

Learn more about the **Frequency Distribution here**.

Solved Example for You

Q: From the given Less than frequency curve, calculate the frequency between 600 and 1000



Sol: By observing the graph, we can see that corresponding values for class intervals between the frequency

600 – 1000 = 16 – 10 = 6

Hence the frequency between said class intervals is 6.

Topic: Measures of Dispersion

Standard Deviation vs. Variance: What's the Difference?

Standard Deviation vs. Variance: An Overview

Standard deviation and variance may be basic mathematical concepts, but they play important roles throughout the financial sector, including the areas of accounting, economics, and investing. In the latter, for example, a firm grasp of the calculation and interpretation of these two measurements is crucial for the creation of an effective trading strategy.

Standard deviation and variance are both determined by using the mean of the group of numbers in question. The mean is the average of a group of numbers, and the variance measures the average degree to which each number is different from the mean. The extent of the variance correlates to the size of the overall range of numbers—meaning the variance is greater when there is a wider range of numbers in the group, and the variance is lesser when there is a narrower range of numbers.

Standard Deviation

Standard deviation is a statistic that looks at how far from the mean a group of numbers is, by using the square root of the variance. The calculation of variance uses squares because it weights outliers more heavily than data very near the mean. This calculation also prevents differences above the mean from canceling out those below, which can sometimes result in a variance of zero.

Standard deviation is calculated as the square root of variance by figuring out the variation between each data point relative to the mean. If the points are further from the mean, there is a higher deviation within the date; if they are closer to the mean, there is a lower deviation. So the more spread out the group of numbers, the higher the standard deviation.

To calculate standard deviation, add up all the data points and divide by the number of data points, calculate the variance for each data point and then find the square root of the variance.

Variance

The variance is the average of the squared differences from the mean. To figure out the variance, first calculate the difference between each point and the mean; then, square and average the results.

For example, if a group of numbers ranges from 1 to 10, it will have a mean of 5.5. If you square and average the difference between each number and the mean, the result is 82.5. To figure out the variance, subtract 82.5 from the mean, which is 5.5 and then divide by N, which is the value of numbers, (in this case 10) minus 1. The result is a variance of about 9.17. Standard deviation is the square root of the variance so that the standard deviation would be about 3.03.

However, because of this squaring, the variance is no longer in the same unit of measurement as the original data. Taking the root of the variance means the standard deviation is restored to the original unit of measure and therefore much easier to measure.

Special Considerations
For traders and analysts, these two concepts are of paramount importance as the standard deviation is used to measure security and market volatility, which in turn plays a large role in creating a profitable trade strategy.

Standard deviation is one of the key methods that analysts, portfolio managers, and advisors use to determine risk. When the group of numbers is closer to the mean, the investment is less risky; when the group of numbers is further from the mean, the investment is of greater risk to a potential purchaser.

Securities that are close to their means are seen as less risky, as they are more likely to continue behaving as such. Securities with large trading ranges that tend to spike or change direction are riskier. In investing, risk in itself is not a bad thing, as the riskier the security, the greater potential for a payout as well as loss. (For related reading, see "What Does Standard Deviation Measure In a Portfolio?")

KEY TAKEAWAYS

- Standard deviation looks at how spread out a group of numbers is from the mean, by looking at the square root of the variance.
- The variance measures the average degree to which each point differs from the mean—the average of all data points.
- The two concepts are useful and significant to traders, who use them to measure market volatility.